Practice Sheet - 9

(APPLICATIONS OF LAPLACE TRANSFORMS)

Course	**:**	**B.Sc. (H) Physics**
Semester	**:**	**IV**
Subject	**:**	**Mathematical Physics III**
UPC	**:**	**32221401**
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1. Solve the following differential equation using the method of Laplace Transforms.

   (i) $y''' - 3y'' + 3y' - y = t^2 e^t$ with $y(0) = 1,\ y'(0) = 0,\ y''(0) = -2$

   (ii) $y'' + y = t$ with $y(0) = 1,\ y'(0) = -2$

   (iii) $y'' - 4y' + 4y = 0$ with $y(0) = 0,\ y'(0) = 3$

   (iv) $y'' + 2y' + 5y = 0$ with $y(0) = 2,\ y'(0) = -4$

   (v) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 2y = 5\sin t$ with $y(0) = y'(0) = 0$

   (vi) $y'' - 3y' + 2y = 4t + 12e^{-t}$ with $y(0) = 6,\ y'(0) = -1$

   (vii) $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1,\ y'(0) = 1$

   (viii) $y'' + 9y = \cos 2t$ with $y(0) = 1,\ y(\pi/2) = -1,\ y'(0) = c$

   (ix) $y'' + a^2 y = F(t)$ with $y'(0) = -2,\ y(0) = 1$

   (x) $4y'' - 4y' + 37y = 0$ with $y(0) = 3,\ y'(0) = 10.5$

2. Using the method of Laplace Transforms, find the solution of the initial value problem

   $$y'' + 9y = 9U(t-3)$$

   where $y(0) = y'(0) = 0$ and $U(t-3)$ is unit step function.

3. Using Laplace Transforms, solve

   (i) $\dfrac{dx}{dt} = 2x - 3y,\quad \dfrac{dy}{dt} = y - 2x$ subject to $x(0) = 8,\ y(0) = 3$

   (ii) $\dot{x} + x + y = 0,\quad \dot{y} + 4x + y = 0$ where $x(0) = y(0) = 1$

   (iii) $\dfrac{dx}{dt} + y = 0,\quad \dfrac{dy}{dt} - x = 0$ where $x(0) = 1,\ y(0) = 0.$

4. Solve: $\dfrac{\partial u(x,t)}{\partial x} + x\dfrac{\partial u(x,t)}{\partial t} = 0$ where $u(x,0) = 0,\ u(0,t) = t$

5. Using Laplace Transforms, solve the heat equation: $\dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{\partial x^2}$

   subject to the condition $u(x,0) = 30\cos 5x,\ u_x(0,t) = \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0,\ u\left(\dfrac{\pi}{2}, t\right) = 0$

6. Using Laplace Transforms, solve: $\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial x^2}$

if $y(0, t) = 0 = y(5, t)$ and $y(x, 0) = 10\sin 4\pi x$

7. Using Laplace Transforms, solve: $\frac{\partial u(x, t)}{\partial x} = 2\frac{\partial u(x, t)}{\partial t} + u(x, t)$

where $u(x, 0) = 6e^{-3x}$ and $u(x, t)$ is bounded for $x > 0$, $t > 0$.

8. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for a rod of unit length with initial condition $u(x, 0) = \sin \pi x$ with the boundary condition that the two ends of the rod are immersed in reservoir at zero temperature.

9. Solve:

(a) $\frac{\partial u}{\partial x} + 2x\frac{\partial u}{\partial t} = 2x$ where $u(x, 0) = 1$, $u(0, t) = 1$

(b) $x\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} = t\,x$ where $u(x, 0) = 0$, $u(0, t) = 0$

10. Consider an LC circuit. At t = 0, emf E_0 is applied for a short time. Find the current in the system at t > 0.