INTERMEDIATE MACROECONOMICS-II

B.A.(H) Economics, Semester-IV

Topic-1: Technological Progress & Elements of Endogenous Growth

Lecture Notes

(Ref: Jones, *Introduction to Economic Growth*, 2^{nd} ed. Ch-4 & 5.)

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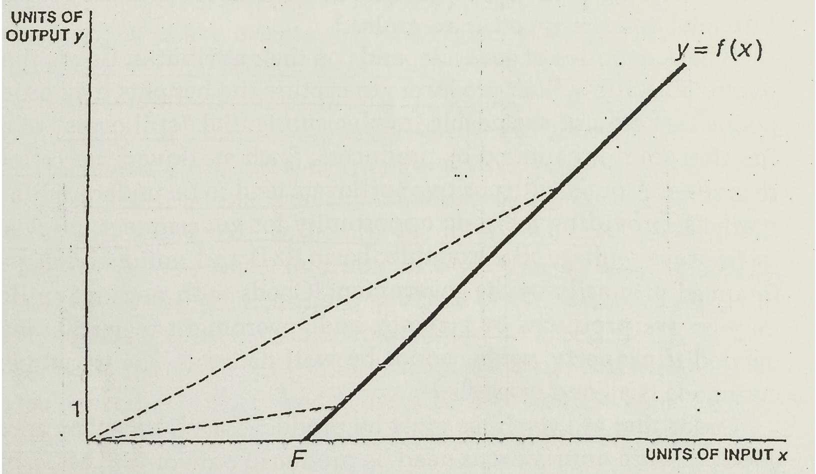
Economics of Ideas

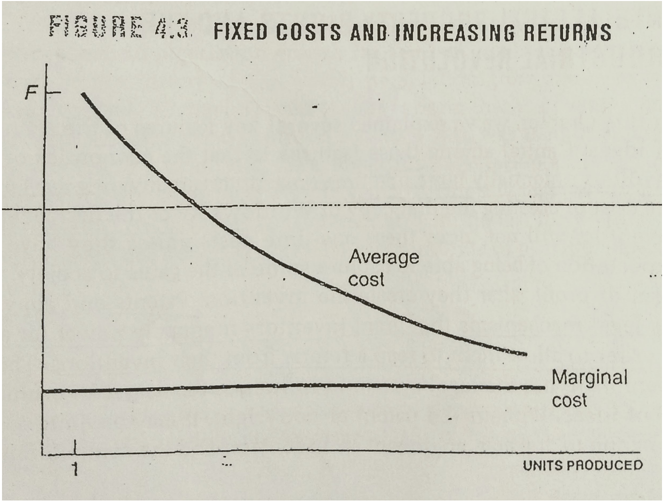
$\boxed{\text{Ideas} \rightarrow \text{Nonrivalry} \rightarrow \text{Increasing Returns} \rightarrow \text{Imperfect Competition}}$

FIXED COSTS AND INCREASING RETURNS

FIGURE 4.2 FIXED COSTS AND INCREASING RETURNS



FIXED COSTS AND INCREASING RETURNS



Engine of Growth

Basic Elements of the Model

$$Y = K^{\alpha}(AL_y)^{1-\alpha} \tag{1}$$

where, K represents capital stock, L_y labor employed in the production, Y represents output, A represents the stock, and $\alpha \in [0, 1]$ is a parameter.

$$\dot{K} = s_K Y - dK. \tag{2}$$

where, s_K rate of savings or rate of forgoing consumption, depreciates at the exogenous rate d, and n represents the rate of growth of population.

$$\frac{\dot{L}}{L} = n \tag{3}$$

$$\text{Number of new ideas produced: } \dot{A} = \bar{\delta} L_A. \tag{4}$$

where, $\bar{\delta}$ represents the rate of discovery of new ideas, and L_A represents the population engaged in research.

$$\text{Rate of discovery: } \bar{\delta} = \delta A^{\phi} \tag{5}$$

where δ and ϕ are constants. $\phi > 0$ represents the productivity of research increases with the stock of ideas that have already been discovered; $\phi < 0$ corresponds to **fishing out**.

$$\dot{A} = \delta L_A^{\lambda} A^{\phi} \tag{6}$$

where, $\lambda \in [0, 1]$ represents the productivity of labor in terms of producing new ideas. Externality associated with ϕ is referred to as **standing on shoulders** effect and the externality associated with λ is referred to as **stepping on toes** effect.

Distribution of total labor force L, into labor engaged in research L_A and labor engaged in production L_Y.

$$L_Y + L_A = L \tag{7}$$

The model assumes that a constant fraction, $\frac{L_A}{L} = s_R$, of the labor force engages in research to produce new ideas, and the remaining fraction, $1 - s_R$, produces output.

Growth in Romer Model

Along a balanced growth path, for rate of growth of per capita output, g_y, rate of growth of capital-labor ration, g_k, and of growth of stock of ideas, g_A.

$$g_y = g_k = g_A \tag{8}$$

And rate of technological progress,

$$\frac{\dot{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}} \tag{9}$$

Along a balanced growth path, as $\dot{A}/A \equiv g_A$ is constant. Therefore,

$$0 = \lambda \frac{\dot{L_A}}{L_A} - (1-\phi)\frac{\dot{A}}{A} \qquad (10)$$

Also, along a balanced growth path, growth rate of number of researches must be equal to growth rate of the population i.e. $\frac{\dot{L_A}}{L_A} = n$. Thus,

$$g_A = \frac{\lambda n}{1-\phi} \qquad (11)$$

Special Examples

Case 1: Let $\lambda = 1$, $\phi = 0$, L_A is a constant, and the productivity of researchers δ is constant. Then

$$\dot{A} = \delta L_A$$

The economy generates a constant number of new ideas $\dot{A} = \delta L_A$, each period. Therefore, the growth rate of stock of ideas $\dot{A}/A$ falls overtime, eventually approaching zero. And, the growth rate of per capita output also falls overtime and economy doesn't operate at a sustainable growth rate even with a constant research effort.

Case 2: Let $\lambda = 1$, $\phi = 1$, L_A is a constant, and the productivity of researchers δ is constant. Then

$$\dot{A} = \delta L_A A$$

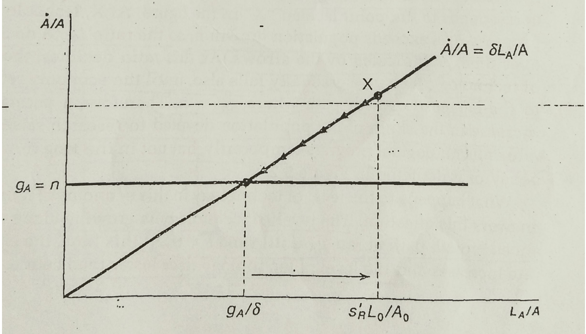
$$\implies \frac{\dot{A}}{A} = \delta L_A$$

This suggests that the growth rate of stock of ideas is constant in each period. Therefore, the growth rate of per capita ouput is also constant and even with constant number of researchers, the economy operate at a sustainable growth rate .

Growth Effect vs Level Effect

Let the share of population engaged in research sector increases permanently. Also, let $\lambda = 1$ and $\phi = 0$. Let s_R increases permanently to s_R'. Also, $\dfrac{\dot{A}}{A} = \delta \dfrac{s_R L}{A}$

FIGURE 5.1 TECHNOLOGICAL PROGRESS: AN INCREASE IN THE R&D SHARE



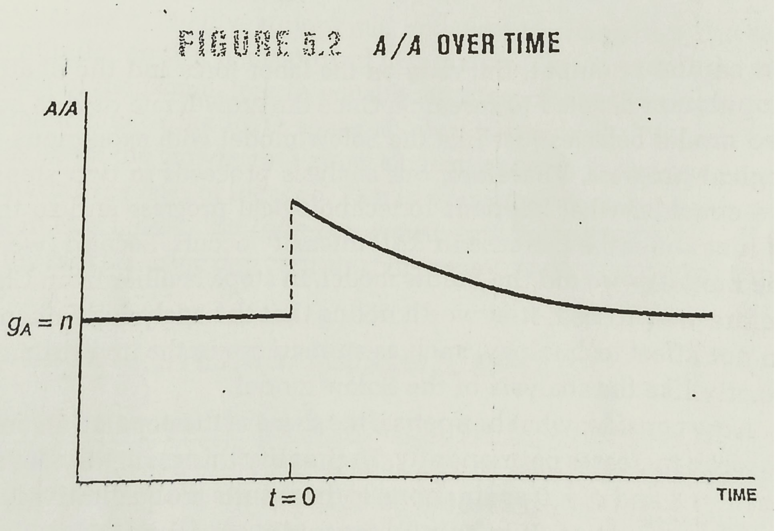
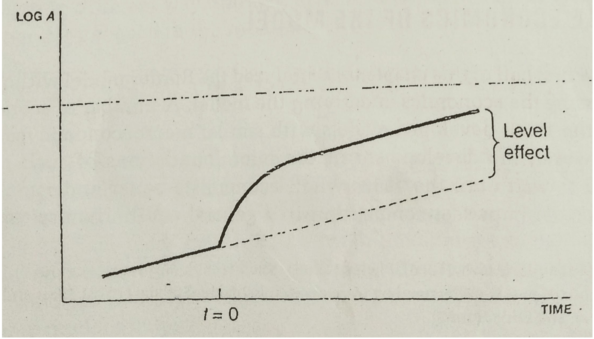


FIGURE 5.2 A/A OVER TIME
A/A
$g_A = n$
$t = 0$
TIME

$$\left(\frac{y}{A}\right)^{*} = \left(\frac{s_K}{n + g_A + d}\right)^{\alpha/1-\alpha}(1 - s_R)$$

$$y^{*}(t) = \left(\frac{s_K}{n + g_A + d}\right)^{\alpha/1-\alpha}(1 - s_R)\frac{\delta s_R}{g_A}L(t)$$



THE LEVEL OF TECHNOLOGY OVER TIME

Economics of the Model

The Final-Goods sector

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha} \tag{12}$$

where, Output Y, is produced using labor, L_Y, a number of different capital goods, X_j, also referred to as intermediate goods, and A measures the number of capital goods.

$$Y = L_Y^{1-\alpha} x_1^{\alpha} + L_Y^{1-\alpha} x_2^{\alpha} + \cdots + L_Y^{1-\alpha} x_A^{\alpha}$$

$$\approx L_Y^{1-\alpha} \int_0^A x_j^{\alpha} dj$$

At wage rate w and rental price of each capital good p_j, firms decide the level of labor and each capital good, in order to maximize the profit function.

$$\max_{L_Y, x_j} \; L_Y^{1-\alpha} \int_0^A x_j^{\alpha} dj - wL_Y - \int_0^A p_j x_j dj$$

From the first-order conditions, we get

$$w = (1-\alpha)\frac{Y}{L_Y} \tag{13}$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} \tag{14}$$

The Intermediate-Goods sector

Consists of monopolists, each produces a particular capital good.
Firms gain monopoly power by purchasing the design for a specific capital good from research sector and produce each unit of capital at a fixed cost of raw material r.
Profit maximization problem for a firm in this sector is

$$\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j$$

where $p_j(x)$ is the demand function for the capital good x.
From the first-order conditions, we get

$$p'(x)x + p(x) - r = 0$$

$$p'(x)\frac{x}{P} + 1 = \frac{r}{P}$$

$$p = \frac{1}{1 + \frac{p'(x)x}{p}} r$$

The elasticity $p'(x)x/p$ can be calculated from the demand function of each capital good, i.e. $p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1}$. And $p'(x)x/p = (\alpha - 1)$. Such that,

$$p = \frac{1}{1 + \frac{p'(x)x}{p}} r = \frac{1}{\alpha} r$$

This suggests that each capital good is sold at the same price. Therefore, by keeping the quantity of each good same i.e. $x_j = x$, we get profit as

$$\pi = \alpha(1-\alpha)\frac{Y}{A}$$

And the total demand for capital goods will be equal to the total capital stock in the economy, K:

$$\int_0^A x_j dj = \sum_{j=1}^{A} x_j = xA = K$$

The final goods production function can be written, using $x_j = x$, as

$$Y = AL_Y^{1-\alpha} x^{\alpha}$$

$$Y = AL_Y^{1-\alpha} A^{-\alpha} K^{\alpha}$$

$$= K^{\alpha} (AL_Y)^{1-\alpha}$$

Research Sector

Let P_A be the price of the new design, $\dot{P}_A$ be the capital gain by reselling the design, r be the prevailing rate of interest in the market, and π be the profit earned.

Then from the *arbitrage equation*, we get

$$rP_A = \pi + \dot{P}_A$$

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}$$

Along a balanced growth path, r is constant. Therefore, π/P_A must also be constant, which requires both π and P_A to grow at the same rate, which turns out to be population growth rate n.

Therefore, we get

$$P_A = \frac{\pi}{r - n}$$

Labor working in the final-good sector earns a wage,

$$w_Y = (1-\alpha)\frac{Y}{L_Y}$$

Labor working in the research sector, based on their productivity $\bar{\delta}$ and price of new design P_A, earns a wage,

$$w_R = \bar{\delta} P_A$$

In the context of labor, being indifferent to engage in either production or research sector, $w_Y = w_R$, which gives,

$$s_R = \frac{1}{1 + \frac{r-n}{\alpha g_A}}$$

- The interest rate in this three sector economy is, $r = \alpha^2 Y/K$, which is less than the marginal product of capital i.e. $\alpha Y/K$.
- So, as oppose to the solow model, under the romer model, as the production in the economy is characterized by the increasing returns and all factors cannot be paid their marginal products.

Optimal R&D

FIGURE 5.4 THE "CONSUMER-SURPLUS EFFECT"