Inference Rules for Functional Dependencies

From the real world you describe based on knowledge and facts some dependencies that describe the database or the problem. Some of the dependencies are not obvious and can be inferred by use of inference rules. Prior to describing inference rules, we must understand the concept of closure that is denoted by the symbol +. So X^+ means closure of X. It also implies the attributes that are dependent on X.

For example, suppose that we specify the following set F of obvious functional dependencies on the relation schema

F = { SSN → [Ename, Bdate, Address, Dnumber

Dnumber → Dname, DMgr_SSN

Some of the FD's that we can infer from F are the following

SSN → [Dname, DMGR_SSN]

SSN → SSN

DNUMBER → DNAME

The closure F^+ of F is the set of all functional dependencies (FD's) that can be inferred from F.

Inference Rules

IR1 (reflexive rule): If $X \supseteq Y$, then $X \rightarrow Y$

IR2 (augmentation rule): $X \rightarrow Y$ implies $XZ \rightarrow YZ$

IR3 (transitive rule): If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

IR4 (projective rule) If $X \rightarrow YZ$ then $X \rightarrow Y$

IR5 (union rule): If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$

IR6 (Pseudotransitive rule) If $X \rightarrow Y$ and $WY \rightarrow Z$ then $WX \rightarrow Z$

Following proof of inference rules may be studied even though these are only for reference:

Proof of IR1: Suppose that $X \supseteq Y$ and two tuples t_1 and t_2 exist in some relation instance r of R such that

$t_1[X] = t_2[X]$. Then

$t_1[Y] = t_2[Y]$ because $X \supseteq Y$

hence, $X \rightarrow Y$ must hold in r.

Proof of IR2 (by contradiction):

Assume that $X \rightarrow Y$ holds in a relation instance r of R but that $XZ \rightarrow YZ$ does not hold. Then there must exist two tuples t_1 and t_2 such that

(1) $t_1[X] = t_2[X]$

(2) $t_1[Y] = t_2[Y]$

(3) $t_1[XZ] = t_2[XZ]$

and (4) $t_1[YZ] \neq t_2[YZ]$

This is not possible because (1) and (3) we deduce

(5) $t_1[Z] = t_2[Z]$

and from (2) and (5) we deduce (6) i.e $t_1[YZ] = t_2[YZ]$, contradicting (4).

Proof of IR3

Assume that (1) $X \to Y$ and (2) $Y \to Z$ both hold in a relation r. Then for any two tuples t_1 and t_2 in r such that $t_1[X] = t_2[X]$ we must have (3)

(3) $t_1[Y] = t_2[Y]$ from assumption (1); hence we must have (4)

$t_1[Z] = t_2[Z]$ from (3) and assumption (2), thus $X \to Z$ must hold in r.

Proof of IR4 (using IR1 and IR3)

1. $X \to YZ$ given
2. $YZ \to Y$ using IR1 and knowing $YZ \supseteq Y$
3. $X \to Y$ (using IR3 on (1) & (2))

Proof of IR5 using IR1 through IR3

1. $X \rightarrow Y$ given
2. $X \rightarrow Z$ given
3. $X \rightarrow XY$ using IR1 on 1 by augmenting with X as $XX = X$
4. $XY \rightarrow YZ$ (using IR2 on 2 by augmenting on Y
5. $X \rightarrow YZ$ (using IR3 on 3 & 4)

Proof of IR6 (using IR1 thru IR3)

1. $X \rightarrow Y$ (given)
2. $WY \rightarrow Z$ (given)
3. $WX \rightarrow$ (WY using IR2 on 1 by augmenting on ~~Y~~W)
4. $WX \rightarrow Z$ (using IR3 on 3 & 2)

The above are also called armstrong rules.