Section 3.5 (Bartle)

Cauchy sequence

Def^n: A sequence (x_n) is said to be a Cauchy s

if the terms of the seq. are ultimately very clo
to each other.

i.e. for every $\epsilon > 0$, $\exists$ a natural no. H (depend upon

such that $|x_n - x_m| < \epsilon \quad \forall\, n, m \geq H$

(Note: here n & m, both are variables & H is Const

Exp. $(x_n) = \left(\frac{1}{n}\right)$

T.S. (x_n) is a Cauchy seq.
we need to show that for $\epsilon > 0$, $\exists\, H \in \mathbb{N}$:

$$|x_n - x_m| < \epsilon \quad \forall\, n, m \geq H$$

i.e. T.S. $\left|\frac{1}{n} - \frac{1}{m}\right| < \epsilon \quad \forall\, n, m \geq H$

Now, $\left|\frac{1}{n} - \frac{1}{m}\right| \leq \frac{1}{n} + \frac{1}{m} < \frac{\epsilon}{2} + \frac{\epsilon}{2}$ if $\frac{1}{n} < \epsilon/2$ & $\frac{1}{m} < \epsilon/2$

i.e. if $n > \frac{2}{\epsilon}$ & $m >$

Let $H \in \mathbb{N}$ be such that $H > 2/\epsilon$

then $\left|\frac{1}{n} - \frac{1}{m}\right| < \epsilon \quad \forall\, n, m \geq H$

$\Rightarrow \left(\frac{1}{n}\right)$ is a Cauchy seq.