21.3 RETARDATION PLATES

When a doubly refracting crystal is cut in the form of a plate in such a way that the section is parallel to the optic axis and the plate is employed to introduce a given phase difference between the ordinary and extraordinary waves while passing through it, it is called a *retardation or phase-retardation plate.* The introduced phase difference can be calculated as follows :

If t be the thikness of the plate in the direction of propagation, μ_0 the refractive index for O-ray and μ_e that for E-ray, then within the plate, the optical paths for O-ray and E ray are $\mu_o t$ and $\mu_e t$ respectively.

The path difference is therefore,

$$\Delta = (\mu_o - \mu_e)t \qquad (\because \mu_o > \mu_e)$$

and the corresponding phase difference between the two waves is given by

$$\delta = \frac{2\pi}{\lambda}(\mu_o - \mu_e)t \qquad ...(1)$$

when a plane polarized light is incident on a retarder, it splits the light into two plane polarized light waves. One wave lags behind the other by certain amount. When they emerge from the retarder, they superpose on each other producing a wave of different state of polarization. A *quarter wave plate* and *half wave plate* are important retarders.

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21.3.1 Quarter-Wave Plate

It is a uniaxial doubly refracting crystal plate, cut with its optic axis parallel to the refracting faces, and can produce a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between the ordinary and the extraordinary rays.

When a beam of plane-polarized monochromatic light of wavelength λ is incident normally on a crystal of this type it is dissociated into O and E components which travel in the *same* direction but with *different* velocities (Fig. 21.3).

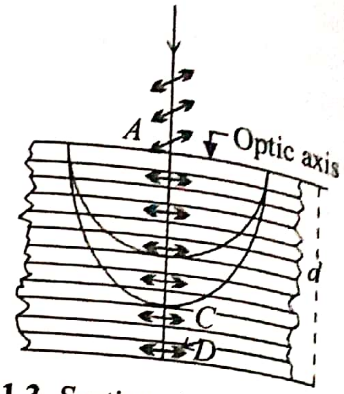

Fig. 21.3. *Section of a negative crystal containing the incident ray and showing separation of E and O waves cut with its optic axis in the plane of incidence and parallel to the refracting surface.*

In the case of *negative* crystal such as calcite E-ray travels faster than O-ray so that $\mu_o > \mu_e$.

If t be the thickness of the plate then the distance travelled in the crystal is equivalent to distances $\mu_o t$ and $\mu_e t$ in air for the O and E rays respectively. Therefore, the resultant path difference between the two components will be

$$= t(\mu_o - \mu_e).$$

Since the phase changes by 2π for one wavelength λ, then to introduce a phase difference of $\pi/2$ between O and E rays is to introduce a path difference of $\lambda/4$, we have

$$\frac{\lambda}{4} = t(\mu_o - \mu_e)$$

or

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

or

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} \qquad ...(2)$$

This is the thickness of a quarter-wave plate.

For a *positive* crystal such as quartz $\mu_e > \mu_o$,

so that

$$t = \frac{\lambda}{4(\mu_e - \mu_o)} \qquad ...(2a)$$

A quarter-wave plate is the simplest device for producing and detecting circularly-polarized light. In conjuction with a nicol prism it is used for analysing all kinds of polarized light.

21.3.2 Half-Wave Plate

A uniaxial doubly refracting crystal, cut with its faces parallel to its optic axis which can introduce a phase difference of π or a path difference of $\lambda/2$ between the O and E rays is called a half-wave plate.

If t be the thickness of such a plate, then for a negative crystal such as calcite $\mu_o > \mu_e$.

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$$\frac{\lambda}{2} = t(\mu_o - \mu_e)$$

$$\therefore \qquad t = \frac{\lambda}{2(\mu_o - \mu_e)}. \qquad \ldots(3)$$

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For a positive crystal such as quartz $\mu_e > \mu_o$

$$\therefore \qquad t = \frac{\lambda}{2(\mu_e - \mu_o)}. \qquad \ldots(3a)$$

Since the path difference depends upon the wavelength of light used, the principal indices for yellow sodium light, $\lambda = 5893 \times 10^{-8}$ cm are generally used for computing the required thickness of a quarter-wave plate or a half-wave plate.

A **half-wave plate** finds its application in the construction of Laurent's half-shade device used in a polarimeter. Such plates are often made of thin sheets of quartz or of split mica cut parallel to the optic axis.

The following cases indicating the action of quarter and half-wave plates are worth-noting.

(*i*) When plane-polarized light is incident on a quarter-wave plate, the emergent light is in general elliptically polarised; the axes of the ellipse are parallel and perpendicular to the optic axis and the ratio of the axes is given by tan θ, where θ is the angle which the plane of polarization of the incident beam makes with the optic axis. When θ is 45°, the light emerging from a quarter-wave plate is circularly polarized.

(*ii*) When a plane-polarized light is incident upon a half-wave plate, the transmitted light is also plane-polarized. If the plane of polarization of the incident light makes an angle θ with the direction of the optic axis, the plane of polarization of the emergent beam makes an angle $-\theta$ with the same direction, i.e., the plane has effectively been rotated through an angle 2θ.

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