

Problems related to Electricity, Magnetism and Electromagnetic Theory

1. What is the physical significance of divergence of a vector field?
(Answer: It represents the net amount of flux coming out of a volume element. It's a scalar quantity. (a) If the $\nabla \cdot \mathbf{F} > 0$ at any point P, then the point is **SOURCE** of the field, (b) If $\nabla \cdot \mathbf{F} < 0$, then the point is **SINK**, (c) If $\nabla \cdot \mathbf{F} = 0$, the field is a solenoidal vector without source or a sink.)

2. Five thousand lines of a electric force enter in a certain region and three thousand lines emerge from it. Find the total charge in coulomb within the region.
(Answer: Net flux diverging = $5000 - 3000 = 2000$. From Gauss's law $\phi = \frac{q}{\epsilon_0}$, $q = 1.77 \times 10^{-8} \text{ C}$.)

3. Can the following be a possible electrostatic field? $\mathbf{E} = xy \mathbf{i} + 2yz \mathbf{j} + 3xz \mathbf{k}$
(Hint: We need to prove $\nabla \times \mathbf{E} = 0$, the above field is not an electrostatic field).

4. State Poynting theorem and explain what do you understand by the Poynting vector.

5. If \mathbf{A} and \mathbf{B} are irrotational, prove that $\mathbf{A} \times \mathbf{B}$ is solenoidal.
(Hint: need to prove $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = 0$, given $\nabla \times \mathbf{A} = 0$ and $\nabla \times \mathbf{B} = 0$.)

6. Distinguish between diamagnetic, paramagnetic and ferromagnetic materials.
(Hint: Can write any 3 important differences. (a) Intrinsic magnetic moment, (b) Susceptibility, (c) M-H plot and transition temperature)

7. Discuss the difference between induced electric field and electric field due to static charges.

(Hint: (a) Explain Faraday's experiments and then write about EM induction process. (b) Conservative nature of electric field)

8. Write the equation of continuity and explain its physical significance.
(Hint: write the equation of continuity and explain the symbols in it. It talks about the Law of conservation of charge. Total current flowing out of some volume must be equal to rate of decrease of charge within the volume assuming that charge can neither be created nor destroyed.)
9. Distinguish between self and mutual inductance.
10. What is the relation between \mathbf{E} , \mathbf{P} , and \mathbf{D} where the symbols have their usual meaning.
(Answer: $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ where \mathbf{D} is the dielectric displacement vector, \mathbf{E} is the Electric field intensity and \mathbf{P} is the Polarization Vector).
11. Find the work done in moving a particle in the force field $\mathbf{F} = (2x+y^2) \mathbf{i} + (3y-4x) \mathbf{j}$, along the straight lines from (0,0) to (2,0), then to (2,1), then to (0,0). (Answer: -4.6).
12. Show that the following function is a sink field $v = \frac{-x \mathbf{i} - y \mathbf{j}}{\sqrt{x^2 + y^2}}$.
(Hint: need to prove divergence of given potential ($\nabla \cdot v < 0$)).
13. What is meant by polarization of a dielectric?
14. Find the energy stored in the magnetic field of a 50 mH coil carrying a current of 2A. (Answer: $E = \frac{1}{2} LI^2 = 0.1 \text{ Joule}$).
15. Calculate the capacitance of a parallel plate capacitor of plate area 5 cm² and separated by dielectric of dielectric constant 4 and thickness 1 cm. (Answer: $C = \frac{k \epsilon_0 A}{d} = 1.74 \times 10^{-12} \text{ F}$).

16. The electric potential at any point (x,y,z) is given by $V = x(3y^2 - x^2 + z)$. Find the electric field at that point.

(Answer: We need to calculate $\mathbf{E} = -\nabla \cdot V = -\mathbf{i}(3y^2 - 3x^2) - \mathbf{j}(6xy) - \mathbf{k}x$).

17. A charge 'q' moving initially with velocity $3\mathbf{k}$ m/s enters a region with electric field, $\mathbf{E} = 10\alpha\mathbf{i}$ V/m and magnetic field, $\mathbf{B} = 20\mathbf{j} + 100\mathbf{k}$ Tesla. For what value of 'α' will be Lorentz force on the charge be zero. (Answer: $\mathbf{F} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B})$, $\alpha = 6$).

18. Find a unit vector normal to the surface $xz^2 + x^2y = z - 1$, at the point (1, -3, 2). (Answer: $\frac{-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}}{\sqrt{14}}$).

19. The magnetic field in a region is given by $\mathbf{B} = 3\mathbf{i} + 4\mathbf{k}$ tesla. Calculate the magnetic flux across the surfaces each of area 2 m^2 in (a) x-y, (b) y-z, (c) z-x planes.

(Answer: $\phi = \mathbf{B} \cdot d\mathbf{s}$; (a) $\phi = (3\mathbf{i} + 4\mathbf{k}) \cdot (2\mathbf{k}) = 8$;

(b) $\phi = (3\mathbf{i} + 4\mathbf{k}) \cdot (2\mathbf{i}) = 6$;

(c) $\phi = (3\mathbf{i} + 4\mathbf{k}) \cdot (2\mathbf{j}) = 0$)

20. Sunlight strikes the earth outside its atmosphere with intensity of $2.0\text{ cal/cm}^2\text{ min}$. Find the peak value of E and B for sunlight at the earth.

(Answer: Poynting vector is energy flux/area. time

Given solar energy received by the earth $= 2\text{ cal/cm}^2\text{ min} = \frac{2 \times 4.18 \times 10^4}{60} = 1400$.

$\mathbf{E}\mathbf{H} = 1400$, but $\frac{E}{H} = \sqrt{\mu_o/\epsilon_o} = 376.72$, gives rise to $\mathbf{E} = 726.2\text{ V/m}$ and $\mathbf{H} = 1.927\text{ Amp/m}$

Amplitude of electric and magnetic field vector is given as $\mathbf{E}_o = E\sqrt{2} = 1027\frac{\text{V}}{\text{m}}$, $\mathbf{H}_o = H\sqrt{2} = 2.725\frac{\text{amp}}{\text{m}}$.)