

Practice Sheet - 5 (LAPLACE TRANSFORM)

Course : B.Sc. (H) 2nd year, IV Sem
Subject : Mathematical Physics III
UPC : 32221401
Teacher : Ms. Bhavna Vidhani
(Deptt. of Physics & Electronics)

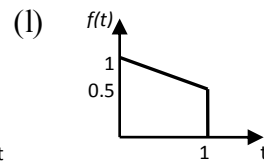
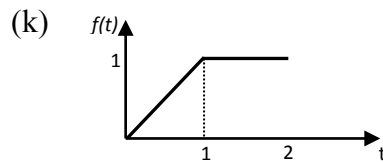
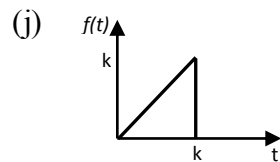
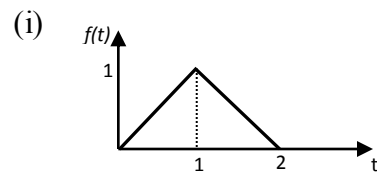
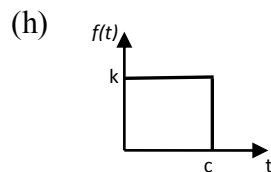
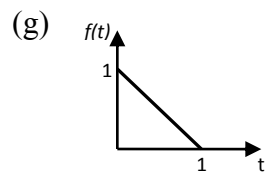
1. Find $L\{f(t)\}$ if $f(t) = \begin{cases} 2t & ; 0 \leq t \leq 5 \\ 1 & ; t > 5 \end{cases}$

2. Find the Laplace Transform of the following functions:

(a) $3t^4 - 2t^3 + 4e^{-3t} - 2\sin 5t + 3\cos 2t$, (b) $e^{-2t} (3\cos 6t - 5\sin 6t)$

(c) $t^2 e^{3t}$ (d) $e^{4t} \cosh 5t$

(e) $\sin(\omega t + \delta)$ (f) $(\sin t - \cos t)^2$



3. Find the Laplace Transform of the function $u(t - a)$ where $u(t)$ is defined as:

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

4. Find $L\{f(t)\}$ if

(a) $f(t) = \begin{cases} \cos(t - \pi/3) & ; t > \pi/3 \\ 0 & ; t < \pi/3 \end{cases}$

(b) $f(t) = \begin{cases} (t-1)^2; & t > 1 \\ 0; & 0 < t < 1 \end{cases}$

5. Given $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1} \frac{1}{s}$, find $L\left\{\frac{\sin at}{t}\right\}$.

6. Show that $L\left\{\frac{\sinh t}{t}\right\} = \frac{1}{2} \ln\left(\frac{s+1}{s-1}\right)$
7. If $f(t) = \begin{cases} t; & 0 < t < 1 \\ 0; & 1 < t < 2 \end{cases}$
Find $L\{f(t)\}$ where $f(t)$ extends periodically with period 2.
8. If $f(t)$ is a periodic function with period T , find its Laplace Transform and use it to evaluate $L\{|\sin \omega t|\}$.
9. Find $L\{f_{\epsilon}(t)\}$, where $f_{\epsilon}(t) = \begin{cases} 1/\epsilon & ; & 0 \leq t \leq \epsilon \\ 0 & ; & t > \epsilon \end{cases}$
Show that $\lim_{\epsilon \rightarrow 0} L\{f_{\epsilon}(t)\} = 1$.
10. Given $L\left\{2\sqrt{\frac{t}{\pi}}\right\} = \frac{1}{s^{3/2}}$, show that $\frac{1}{s^{1/2}} = L\left\{\sqrt{\frac{1}{\pi t}}\right\}$.
11. Find the Laplace Transform of the following functions:
- | | | | |
|---------------------|-----------------------|------------------|-----------------------|
| (a) $tu(t-1)$ | (b) $t^2 e^{3t}$ | (c) t^2 | (d) $t^2 \cosh \pi t$ |
| (e) $\sin \omega t$ | (f) $t e^{-t} \cos t$ | (g) $1 - e^{-t}$ | (h) $t \cos \omega t$ |
| (i) $t e^t$ | (j) $t^2 \sin 2t$ | | |
12. Evaluate the following integrals using Laplace transform:
- | | | |
|---|---|--|
| (i) $\int_0^{\infty} t e^{-2t} \cos t \, dt$ | (ii) $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t} \, dt$ | (iii) $\int_0^{\infty} \cos x^2 \, dx$ |
| (iv) $\int_0^{\infty} \frac{\sin^2 t}{t^2} \, dt$ | (v) $\int_0^{\infty} \frac{1 - e^{-3u}}{u} \, du$ | (vi) $\int_0^{\infty} \frac{e^{-t} \sin t}{t} \, dt$ |
| (vii) $\int_0^{\infty} e^{-x^2} \, dx$ | | |