Conservation laws

1. Energy & Momentum

All decay and reactions obey the conservation of relativistic total Energy and momentum (derivations will not be done in detail.).

Case 1. $\underset{\text{(rest)}}{A} \xrightarrow{\text{decay}} B + C$

$$Q = (m_i - m_f)c^2 = (m_A - m_B - m_C)c^2$$

energy conservation

$$E_i = m_A c^2$$

$$E_f \Rightarrow E_B + E_C = \sqrt{p_B^2c^2 + m_B^2c^4} + \sqrt{p_C^2c^2 + m_C^2c^4}$$

momentum conservation $p_B = p_C$

solving for T_B & T_C.

Then
$$T_B = \frac{(m_Ac^2 - m_Bc^2 - m_Cc^2)(m_Ac^2 - m_Bc^2 + m_Cc^2)}{2m_Ac^2}$$

$$T_C = \frac{(m_Ac^2 - m_Bc^2 - m_Cc^2)(m_Ac^2 - m_Cc^2 + m_Bc^2)}{2m_Ac^2}$$

Case 2. $\underset{\text{(incident on B)}}{A} + \underset{\text{rest}}{B} \longrightarrow C + D + E + \dots$

min threshold laboratory KE required for A to produce C + D + E + ... when it is incident on B.

$$T_{A_{th}} = \frac{(-Q)(\text{total mass of all initial and final particles})}{2 \times \text{mass of target}}$$

Example:

$$p + p \rightarrow p + p + p + \bar{p}$$

$$Q = -2m_p c^2$$

$$T_{Am} \Rightarrow 2m_pc^2 \times \frac{6m_pc^2}{2m_pc^2} = 6m_pc^2 = 5.63\ GeV$$

mean proton of KE 5.63 GeV incident on a proton will produce 3 proton and 1 antiproton.

2. Angular Momentum

$$A \rightarrow B + C$$

$$|S_A| = |S_B + S_C + l_{B,C}|$$

S = intrinsic spin of the particle
$l_{B,C}$ = relative orbital angular momentum of the BC combination.

$$A \rightarrow B + C + D$$

$$|S_A| = |S_B + S_C + l_{B,C} + S_D + l_{BC,D}|$$

$$A + B \rightarrow C + D$$

$$|S_A + S_B + l_{A,B}| = |S_C + S_D + l_{C,D}|$$

3. Parity

$$A \rightarrow B + C$$

$$\pi_A = \pi_B \cdot \pi_C \cdot (-1)^{l_{B,C}}$$

Parity (π) is absolutely conserved in all strong or electromagnetic interactions, but is violated in weak interactions.

Example. $\phi \rightarrow K^+ + K^-$

$\pi_\phi = ?$; $\pi_{K^+} = \pi_{K^-}$ = may be +1 or −1

★ 1) particle & antiparticle have same parity if they have integral spin.

2) particle & antiparticles have opposite parity if they have half integral spin.

so. $\pi_\phi = \pi_{K^+} \pi_{K^-} (-1)^l$

$\pi_{K^+} \cdot \pi_{K^-} = +1$ always. due to (1)

spin $S_{K^+} = S_{K^-} = 0.$ (we know that)

$S_\phi = 1$

So from angular momentum conservation.

$|S_\phi| = |S_K + S_{K^-} + l_{K^+, K^-}|$

$1 = 0 + 0 + ?$

means that $l_{K^+, K^-} = 1$

hence $\pi_\phi = (-1)^l = (-1)^1 = -1$

parity of ϕ is odd.

4. Baryon Number

This is empirical number serve to help the explaination of observation and nonobservation of some process.
Baryon no. is denoted by B.
Total Baryon no. is conserved in all decays & reactions.
Baryon no. is conserved in all ie strong, weak, em interaction (it may violate but the time scale is very long for unified model).

Now how we define or assign Baryon no.

$B = +1$ to all baryon (ex. n, p, Δ)
$B = -1$ to all antibaryons. (ex $\bar{n}, \bar{p}, \bar{\Delta}$).
$B = 0$ to all non-baryons (mesons, leptons & field particles)

Example.

$$p + p \longrightarrow p + p + p + \bar{p}.$$

$B = +1 + 1 = +2$ for $p + p$.

$B = +1 + 1 + 1 - 1 = +2$ for $p + p + p + \bar{p}$

this reaction is possible. for $\bar{p}$ = $B = -1$

$$\bar{\Lambda}^0 \longrightarrow p + \pi^-$$

$B = -1$ for $\bar{\Lambda}^0$ ($\bar{\Lambda} \rightarrow$ antibaryon.)
$B = +1 + 0$ for $p + \pi^-$ ($\pi \rightarrow$ meson)
So this reaction is not possible or forbidden

due to Baryon no. conservation.

5. Lepton Number

Lepton no. is denoted by: L

Total lepton no. is conserved seperately for each type (e, μ, τ) lepton in all the reaction (ν_e, ν_μ, ν_τ) and decay process.

Lepton no. is conserved in all ie strong, weak, em interactions.

Now we define or assign lepton no.

$L = +1$ for lepton $\begin{cases} \text{ie. } L_e = +1 \text{ for } e^- \\ L_\mu = +1 \text{ for } \mu^- \\ L_\tau = +1 \text{ for } \tau^- \end{cases}$

$L = -1$ for antilepton $\begin{cases} \text{ie. } L_e = -1 \text{ for } e^+ \\ L_\mu = -1 \text{ for } \mu^+ \\ L_\tau = -1 \text{ for } \tau^+ \end{cases}$

$L = 0$ for all non lepton (Hadron + field particles.)

Example.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

L_e $\quad 0 \rightarrow +1 - 1 + 0 = 0.$

L_μ $\quad +1 \rightarrow 0 + 0 + 1 = 1$

this reaction is ~~not possible~~ possible due to conservation of lepton no.

$$\mu \rightarrow e + \gamma$$ (reaction/decay is not possible)

L_e $\quad 0 \rightarrow 1 + 0$ $\quad$ not conserved.

L_μ $\quad 1 \rightarrow 0 + 0$ $\quad$ not conserved.

6. Isospin (we have discussed earlier during nuclear reaction).

Hadrons with similar properties and nearly identical masses are grouped into isospin multiplets.

Exp. (n, p) ; $(\Sigma^+, \Sigma^0, \Sigma^-)$; (π^+, π^0, π^-) ; $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$; and list is so long.

- T_3 (3-rd component of isospin) is defined as

$$T_3 = \frac{Q}{e} - \frac{\bar{Q}}{e} \quad \text{--- (1)}$$

Q - electric charge of a particle
$\bar{Q}$ - average charge of the multiplet
e - electronic charge.

now we will calculate for some hadrons.

- Δ multiplets have average charge

$$\bar{Q} = \frac{2e + 1e + 0e - e}{4} = \frac{2e}{4} = \frac{e}{2}$$

(add the charges of all the members and dividing it with no. of members)

using (1) $T_3(\Delta^{++}) = \frac{2e}{e} - \frac{e/2}{e}$

$= 2 - \frac{1}{2} = +3/2$

$T_3(\Delta^+) = \frac{1e}{e} - \frac{e/2}{e} = 1 - \frac{1}{2} = +1/2$

$T_3(\Delta^0) = \frac{0e}{e} - \frac{e/2}{e} \Rightarrow -1/2$

$T_3(\Delta^-) = \frac{-1e}{e} - \frac{e/2}{e} \Rightarrow -3/2$

value of isospin $T = |T_3|_{max} \Rightarrow 3/2$ for all the Δ's.

using (1) we also calculate T_3 for n & p.

$$T_3(p) = \frac{1e}{e} - \frac{1/2 e}{e}$$

$$\bar{Q} = \frac{1e + 0e}{2} = e/2$$

$$\Rightarrow +1/2$$

$$T_3(n) = \frac{0e}{e} - \frac{1/2 e}{e} = \frac{-1}{2}$$

T_3 is based on the charges of the multiplets. It means conservation of T_3 or T is equivalent to conservation of charges.

T is conserved in strong interaction
T may ~~be~~ violate in weak & em interactions.

Example.

$\eta' \longrightarrow \eta + \pi^0$ (forbidden)

$\eta' \longrightarrow \eta + 2\pi$ (allowed).

$T(\eta', \eta) = 0 \quad T(\pi) = 1$

2nd is allowed because two π having T = 1 ~~and~~ can coupled to form a state with T = 0.

i.e. 1+1 = 2 or
1−1 = 0 ✓

7. Strangeness & charm

Some decay processes are slowed to a weak interaction time 10^{-10} sec.

strong interacting time 10^{-22} sec

This slowing of reaction is accounted by the assignment of strangeness quantum no. S.

Strangeness is absolutely conserved in strong and em interacting process but can change by one unit ($\Delta S = \pm 1$) in weak decays.

S can be found for any given hadron by following eq.

$$S = -(n_s - n_{\bar{s}})$$

n_s = no. of strange quark in the hadron.

$n_{\bar{s}}$ = no. of anti strange quark in the hadron

Similar rules for conservation are followed for charm quantum no.

C can be found for any given hadron as.

$$C = +(n_c - n_{\bar{c}})$$

n_c = no. of charm quark

$n_{\bar{c}}$ = no. of anti charm quark.

Example.

$$D \rightarrow K + \pi$$

D - meson with charm quark.

K - meson with strange quark.

π = meson no. strange or charm quark.

So above ~~se~~ decay is possible by weak interaction because ΔS = change by one unit = +1
ΔC = change by one unit = +1

So Now we can verify that any given decay or process is possible or not by above conservation rules. Mostly we have to check Baryon no.; lepton no.; Isospin ; strangeness only.
Energy, angular momentum & parity are checked if it is asked ~~d~~ to do so.

Gell-Mann-Nishijima formula:
The quantum no. assigned so far for hadrons are not all independent; they are related by following eq.

$$\frac{Q}{e} = T_3 + \frac{B+S}{2}$$

$$\frac{Q}{e} = T_3 + \frac{B+S+C}{2}$$ (charm is added here)