

Conservation laws

1. Energy & Momentum

All decay and reactions obey the conservation of relativistic total Energy and momentum (derivations will ~~not~~ be done in detail.).

Case 1. $A \xrightarrow{\text{decay}} B + C$
(rest)

$$Q = (m_i - m_f) c^2 = (m_A - m_B - m_C) c^2$$

Energy conservation $E_i = m_A c^2$
 $E_f \Rightarrow E_B + E_C = \sqrt{p_B^2 c^2 + m_B^2 c^4} + \sqrt{p_C^2 c^2 + m_C^2 c^4}$

Momentum conservation $p_B = p_C$

Solving for T_B & T_C .

Then $T_B = \frac{(m_A c^2 - m_B c^2 - m_C c^2)(m_A c^2 - m_B c^2 + m_C c^2)}{2 m_A c^2}$

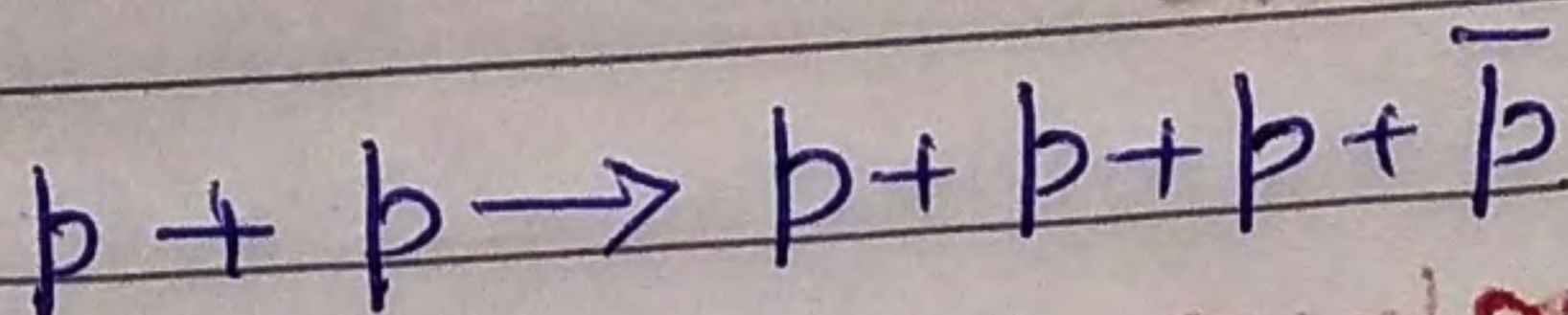
$T_C = \frac{(m_A c^2 - m_B c^2 - m_C c^2)(m_A c^2 - m_C c^2 + m_B c^2)}{2 m_A c^2}$

Case 2. $A + B \rightarrow C + D + E + \dots$
(incident on B) rest

min threshold laboratory KE required for A to produce $C + D + E + \dots$ when it is incident on B.

$$T_{A_{th}} = \frac{(-Q)(\text{total mass of all initial and final particles})}{2 \times \text{mass of target}}$$

Example:

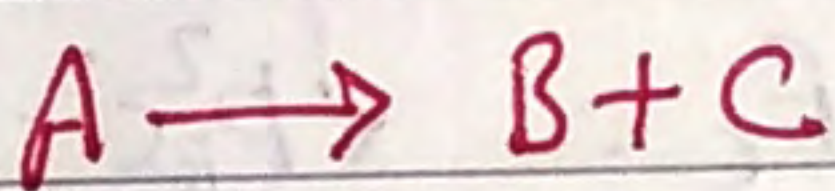


$$Q = -2m_p c^2$$

$$T_{\text{Am}} \Rightarrow 2m_p c^2 \times \frac{6m_p c^2}{2m_p c^2} = 6m_p c^2 = 5.63 \text{ GeV}$$

mean proton of KE 5.63 GeV incident on a proton will produce 3 protons and 1 antiproton.

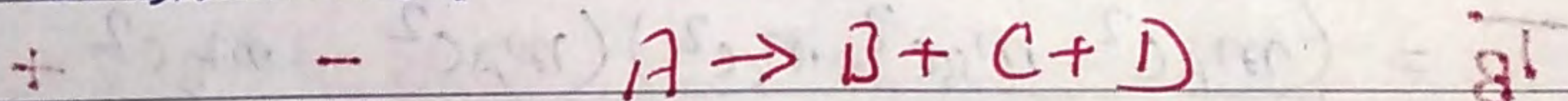
2. Angular Momentum



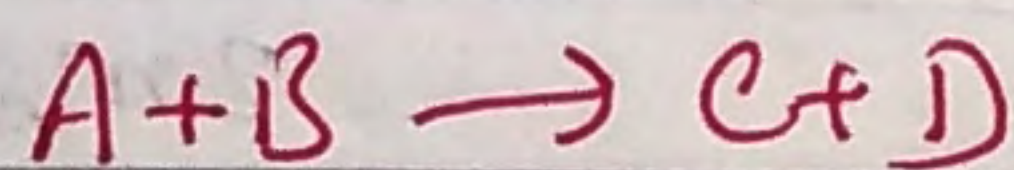
$$|S_A| = |S_B + S_C + l_{B,C}|$$

S - intrinsic spin of the particle

$l_{B,C}$ = relative orbital angular momentum of the BC combination.

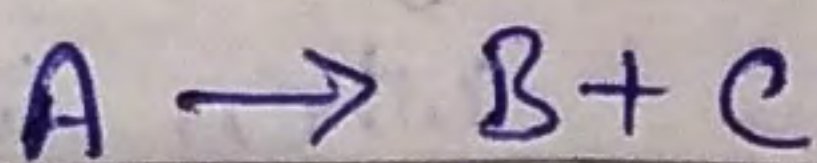


$$|S_A| = |S_B + S_C + l_{B,C} + S_D + l_{B,C,D}|$$



$$|S_A + S_B + l_{A,B}| = |S_C + S_D + l_{C,D}|$$

3. Parity



$$\pi_A = \pi_B \cdot \pi_C \cdot (-1)^{l_{B,C}}$$

Parity (π) is absolutely conserved in all strong or electromagnetic interactions, but is violated in weak interactions.

Example. $\phi \rightarrow K^+ + K^-$

$\pi_\phi = ?$; $\pi_{K^+} = \pi_{K^-} = \text{may be } +1 \text{ or } -1$

- * 1) particle & antiparticle have same parity if they have integral spin.
- 2) particle & antiparticles have opposite parity if they have half integral spins.

so $\pi_\phi = \pi_{K^+} \pi_{K^-} (-1)^l$

$\pi_{K^+} \cdot \pi_{K^-} = +1$ always. due to (1)

spin $S_{K^+} = S_{K^-} = 0$. (we know that)
 $S_\phi = 1$

So from angular momentum conservation

$|S_\phi| = |S_K + S_{K^-} + L_{K^+, K^-}|$

$1 = 0 + 0 + ?$

means that $L_{K^+, K^-} = 1$

hence $\pi_\phi = (-1)^l = (-1)^1 = -1$

parity of ϕ is odd.

4. Baryon Number

This is empirical number serve to help the explanation of observation and non observation of some process.

Baryon no. is denoted by B .

Total Baryon no. is conserved in all decays & reactions.

Baryon no. is conserved in all ie strong, weak, em interaction (it may violate but the time scale is very long for unified model).

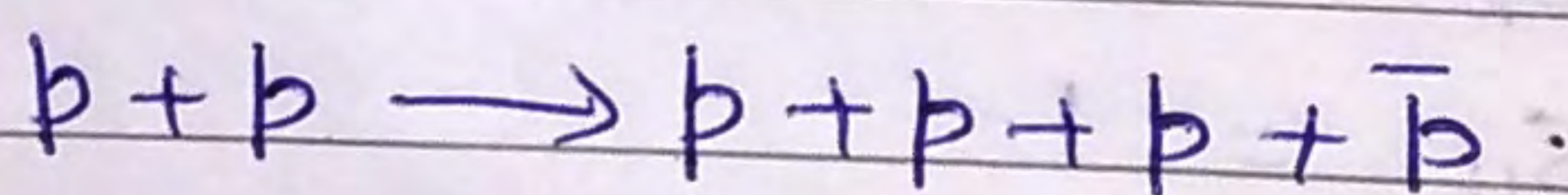
Now how we define ^{or assign} Baryon no.

$B = +1$ to all baryon (ex. n, p, Δ)

$B = -1$ to all antibaryons (ex. $\bar{n}, \bar{p}, \bar{\Delta}$)

$B = 0$ to all non-baryons (mesons, leptons & field particles)

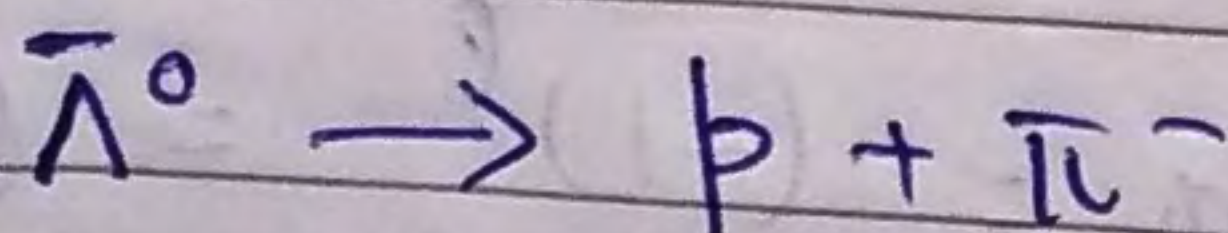
Example.



$$B = +1 + 1 = +2 \text{ for } p + p$$

$$B = +1 + 1 + 1 - 1 = +2 \text{ for } p + p + p + \bar{p}$$

this reaction is possible. for $\bar{p} \Rightarrow B = -1$



$B = -1$ for $\bar{\Lambda}^0$ ($\bar{\Lambda}^0 \rightarrow$ antibaryon.)

$B = +1 + 0$ for $p + \pi^-$ ($\pi^- \rightarrow$ meson)

So this reaction is not possible or forbidden

due to Baryon no. conservation.

5. Lepton Number

Lepton no. is denoted by: L

Total lepton no. is conserved separately for each type (e, μ, τ) lepton in all the reactions ν_e, ν_μ, ν_τ

and decay process.

Lepton no. is conserved in all ie strong, weak, em interactions.

Now we define or assign lepton no.,

$$L = +1 \text{ for lepton } \left\{ \begin{array}{l} \text{ie. } L_e = +1 \text{ for } e^- \\ L_\mu = +1 \text{ for } \mu^- \\ L_\tau = +1 \text{ for } \tau^- \end{array} \right\}$$

$$L = -1 \text{ for anti lepton } \left\{ \begin{array}{l} \text{ie. } L_e = -1 \text{ for } e^+ \\ L_\mu = -1 \text{ for } \mu^+ \\ L_\tau = -1 \text{ for } \tau^+ \end{array} \right\}$$

$$L = 0 \text{ for all non lepton (Hadron + field particles.)}$$

Example.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$L_e \quad 0 \rightarrow +1 - 1 + 0 = 0.$$

$$L_\mu \quad +1 \rightarrow 0 + 0 + 1 = 1$$

this reaction is ~~not possible~~ possible due to conservation of lepton no.

$$\mu \rightarrow e + \gamma$$

(reaction/decay is not possible)

$$L_e \quad 0 \rightarrow 1 + 0$$

not conserved.

$$L_\mu \quad 1 \rightarrow 0 + 0$$

not conserved.

6. Isospin (we have discussed earlier during nuclear reaction)

Hadrons with similar properties and nearly identical masses are grouped into isospin multiplets.

Exp. (n, p) ; $(\Sigma^+, \Sigma^0, \Sigma^-)$; (π^+, π^0, π^-) ; $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$; and list is so long.

T_3 (3-rd component of isospin) is defined as

$$T_3 = \frac{Q}{e} - \frac{\bar{Q}}{e} \quad \text{--- (1)}$$

Q - electric charge of a particle

$\bar{Q}$ - average charge of the multiplet

e - electronic charge.

now we will calculate for some hadrons.

Δ multiplets have average charge

$$\bar{Q} = \frac{2e + 1e + 0e + -e}{4} \quad \text{(add the charges of all the members and dividing it with no. of members)}$$

$$= \frac{2e}{4}$$

$$= \frac{e}{2}$$

$$\text{using (1)} \quad T_3(\Delta^{++}) = \frac{2e}{e} - \frac{e/2}{e}$$

$$= 2 - \frac{1}{2} = +3/2$$

$$T_3(\Delta^+) = \frac{1e}{e} - \frac{e/2}{e} = 1 - \frac{1}{2} = +1/2$$

$$T_3(\Delta^0) = \frac{0e}{e} - \frac{e/2}{e} = -1/2$$

$$T_3(\Delta^-) = \frac{-1e}{e} - \frac{e/2}{e} = -3/2$$

value of isospin $T = |T_3|_{\max} \Rightarrow 3/2$ for all the Δ 's.

using (1) we also calculate T_3 for n & p .

$$T_3(p) = \frac{1e}{e} - \frac{1/2 e}{e}$$

$$\bar{Q} = \frac{1e + 0e}{2} = e/2$$

$$\Rightarrow +1/2 =$$

$$T_3(n) = \frac{0e}{e} - \frac{1/2 e}{e} = -\frac{1}{2} =$$

T_3 is based on the charges of the multiplets. It means conservation of T_3 or T is equivalent to conservation of charges.

T is conserved in strong interaction

T may be violated in weak & em interactions.

Example

$$\eta' \rightarrow \eta + \pi^0 \quad (\text{forbidden})$$

$$\eta' \rightarrow \eta + 2\pi \quad (\text{allowed})$$

$$T_3(\eta', \eta) = 0 \quad T_3(\pi) = 1$$

2nd is allowed because two π having $T=1$ can couple to form a state with $T=0$.

i.e

$$1+1=2 \quad \text{or}$$

$$1-1=0 \quad \checkmark$$

7. Strangeness & charm

Some decay processes are ~~much~~ slowed to a weak interaction time 10^{-10} sec.

strong interacting time 10^{-22} sec

This slowing of reaction is accounted by the assignment of strangeness quantum no. S .

Strangness is absolutely conserved in strong and em interacting process but can change by one unit ($\Delta S = \pm 1$) in weak decays.

~~Strangness~~ S can be found for any given elementary hadron by following eq.

$$S = -(n_s - n_{\bar{s}}) \quad \left\{ \begin{array}{l} n_s = \text{no. of strange quark in the hadron.} \\ n_{\bar{s}} = \text{no. of anti strange quark in the hadron} \end{array} \right.$$

for conservation

Similar rules are followed for charm quantum no.

C can be found for any given hadron as.

$$C = +(n_c - n_{\bar{c}}) \quad \left\{ \begin{array}{l} n_c = \text{no. of charm quark} \\ n_{\bar{c}} = \text{no. of anti charm quark} \end{array} \right.$$

Example.

$$D \rightarrow K + \pi$$

D - meson with charm quantum quark.

K - meson with stray quark.

π = meson no. strange or charm quark.

So above ~~the~~ decay is possible by weak interaction because
 $\Delta S = \text{change by one unit} = +1$
 $\Delta C = \text{change by one unit} = +1$

So Now we can verify that any given decay or process is possible or not by above conservation rules. Mostly we have to check Baryon no.; lepton no.; Isospin; Strangeness only. Energy, angular momentum & parity are checked if it is asked to do so.

Gell-Mann - Nishijima formula:

The quantum no. assigned so far for hadrons are not all independent; they are related by following eq.

$$\frac{Q}{e} = T_3 + \frac{B+S}{2}$$

$$\frac{Q}{e} = T_3 + \frac{B+S+C}{2} \quad (\text{charm is added here})$$