Looking at the equilibrium points, we see that there is no eq. position where $X \neq 0, Y \neq 0, Z \neq 0$ so it is not possible for all three populations to coexist in equillibrium. One predator population has to go extinct in equilibrium

(b). Introducing an extra predator causes either this new predator or the existing predator to become extinct.

two predator species

For equillibrium positions,

$$\frac{dx}{dt} = \frac{dY}{dt} = \frac{dz}{dt} = 0$$

$\therefore$ $a_1X - b_1XY - c_1XZ = 0 \Rightarrow X = 0$ or $a_1 - b_1Y - c_1Z =$

$a_2XY - b_2Y = 0 \Rightarrow Y = 0$ or $X = \frac{b_2}{a_2}$

$a_3XZ - b_3Z = 0 \Rightarrow Z = 0$ or $X = \frac{b_3}{a_3}$

$\therefore$ Possible eq. positions are

$(0,0,0)$, $\left(\frac{b_2}{a_2}, \frac{a_1}{b_1}, 0\right)$, $\left(\frac{b_3}{a_3}, 0, \frac{a_1}{c_1}\right)$

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Q6.9 (a) $\frac{dX}{dt} = a_1X - b_1XY - c_1XZ$

$$\frac{dY}{dt} = a_2XY - b_2Y$$

$$\frac{dZ}{dt} = a_3XZ - b_3Z$$

$X(t)$: prey density

$Y(t)$, $Z(t)$: two predator species

For equilibrium positions,

$$\frac{dX}{dt} = \frac{dY}{dt} = \frac{dZ}{dt} = 0$$