8.5 MAGNETIZATION IN MATERIALS

Our discussion here will parallel that on polarization of materials in an electric field. We shall assume that our atomic model is that of an electron orbiting about a positive nucleus.

We know that a given material is composed of atoms. Each atom may be regarded as consisting of electrons orbiting about a central positive nucleus; the electrons also rotate (or spin) about their own axes. Thus an internal magnetic field is produced by electrons orbiting around the nucleus as in Figure 8.10(a) or electrons spinning as in Figure 8.10(b). Both these electronic motions produce internal magnetic fields $\mathbf{B}_i$ that are similar to the magnetic field produced by a current loop of Figure 8.11. The equivalent current loop has a magnetic moment of $\mathbf{m} = I_b S \mathbf{a}_n$, where S is the area of the loop and I_b is the bound current (bound to the atom).

Without an external **B** field applied to the material, the sum of **m**'s is zero due to random orientation as in Figure 8.12(a). When an external **B** field is applied, the magnetic moments of the electrons more or less align themselves with **B** so that the net magnetic moment is not zero, as illustrated in Figure 8.12(b).

The **magnetization M**, in amperes per meter, is the magnetic dipole moment per unit volume.

If there are N atoms in a given volume Δv and the kth atom has a magnetic moment $\mathbf{m}_k$,

$$\mathbf{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} \mathbf{m}_k}{\Delta v} \tag{8.27}$$

A medium for which **M** is not zero everywhere is said to be magnetized. For a differential volume dv', the magnetic moment is $d\mathbf{m} = \mathbf{M}\, dv'$. From eq. (8.21b), the vector magnetic potential due to $d\mathbf{m}$ is

$$d\mathbf{A} = \frac{\mu_o \mathbf{M} \times \mathbf{a}_R}{4\pi R^2}\, dv' = \frac{\mu_o \mathbf{M} \times \mathbf{R}}{4\pi R^3}\, dv'$$

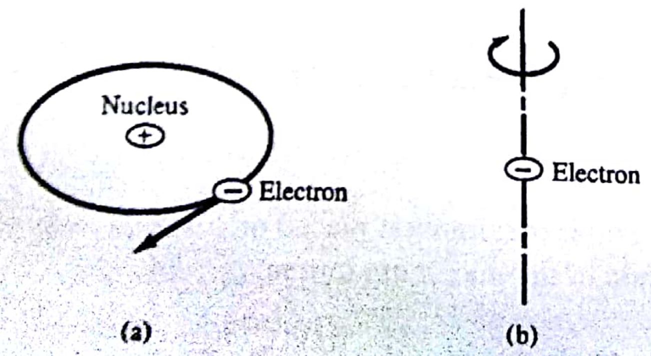

Figure 8.10 (a) Electron orbiting around the nucleus. (b) Electron spin.

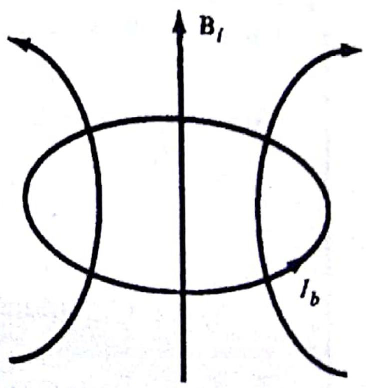

Figure 8.11 Circular current loop equivalent to electronic motion of Figure 8.10.

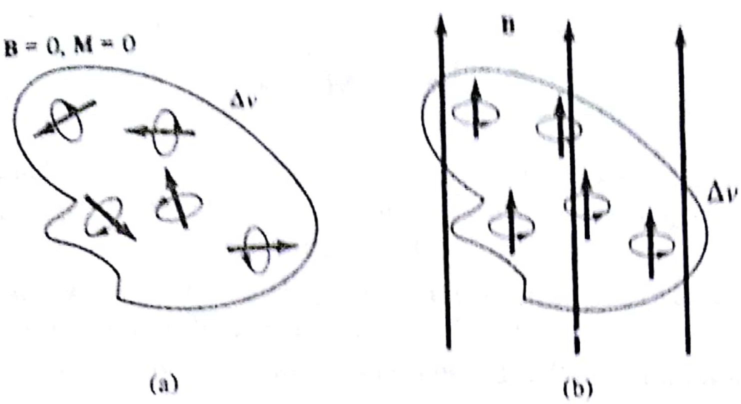

Figure 8.12 Magnetic dipole moment in a volume Δv: (a) before **B** is applied, (b) after **B** is applied.

From to eq. (7.46) we can write

$$\frac{\mathbf{R}}{R^3} = \nabla' \frac{1}{R}$$

Hence,

$$\mathbf{A} = \frac{\mu_o}{4\pi} \int \mathbf{M} \times \nabla' \frac{1}{R}\, dv' \tag{8.28}$$

Using eq. (7.48) gives

$$\mathbf{M} \times \nabla' \frac{1}{R} = \frac{1}{R} \nabla' \times \mathbf{M} - \nabla' \times \frac{\mathbf{M}}{R}$$

Substituting this into eq. (8.28) yields

$$\mathbf{A} = \frac{\mu_o}{4\pi} \int_{v'} \frac{\nabla' \times \mathbf{M}}{R}\, dv' - \frac{\mu_o}{4\pi} \int_{v'} \nabla' \times \frac{\mathbf{M}}{R}\, dv'$$

Applying the vector identity

$$\int_{v'} \nabla' \times \mathbf{F}\, dv' = -\oint_{S'} \mathbf{F} \times d\mathbf{S}$$

to the second integral, we obtain

$$\begin{aligned} \mathbf{A} &= \frac{\mu_o}{4\pi} \int_{v'} \frac{\nabla' \times \mathbf{M}}{R}\, dv' + \frac{\mu_o}{4\pi} \oint_{S'} \frac{\mathbf{M} \times \mathbf{a}_n}{R}\, dS' \\ &= \frac{\mu_o}{4\pi} \int_{v'} \frac{\mathbf{J}_b\, dv'}{R} + \frac{\mu_o}{4\pi} \oint_{S'} \frac{\mathbf{K}_b\, dS'}{R} \end{aligned} \tag{8.29}$$

Comparing eq. (8.29) with eqs. (7.42) and (7.43) (upon dropping the primes) gives

$$\boxed{\mathbf{J}_b = \nabla \times \mathbf{M}} \tag{8.30}$$

and

$$\mathbf{K}_b = \mathbf{M} \times \mathbf{a}_n \tag{8.31}$$

where $\mathbf{J}_b$ is the *bound volume current density* or *magnetization volume current density*, in amperes per meter squared, $\mathbf{K}_b$ is the *bound surface current density*, in amperes per meter, and $\mathbf{a}_n$ is a unit vector normal to the surface. Equation (8.29) shows that the potential of a magnetic body is due to a volume current density $\mathbf{J}_b$ throughout the body and a surface current $\mathbf{K}_b$ on the surface of the body. The vector $\mathbf{M}$ is analogous to the polarization $\mathbf{P}$ in dielectrics and is sometimes called the *magnetic polarization density* of the medium. In another sense, $\mathbf{M}$ is analogous to $\mathbf{H}$ and they both have the same units. In this respect, as $\mathbf{J} = \nabla \times \mathbf{H}$, so $\mathbf{J}_b = \nabla \times \mathbf{M}$. Also, $\mathbf{J}_b$ and $\mathbf{K}_b$ for a magnetized body are similar to ρ_{pv} and ρ_{ps} for a polarized body. As is evident in eqs. (8.29) to (8.31), $\mathbf{J}_b$ and $\mathbf{K}_b$ can be derived from $\mathbf{M}$; therefore, $\mathbf{J}_b$ and $\mathbf{K}_b$ are not commonly used.

In free space, $\mathbf{M} = 0$ and we have

$$\nabla \times \mathbf{H} = \mathbf{J}_f \quad \text{or} \quad \nabla \times \left(\frac{\mathbf{B}}{\mu_o}\right) = \mathbf{J}_f \tag{8.32}$$

where $\mathbf{J}_f$ is the free current volume density. In a material medium $\mathbf{M} \neq 0$, and as a result, $\mathbf{B}$ changes so that

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_o}\right) = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}$$
$$= \nabla \times \mathbf{H} + \nabla \times \mathbf{M}$$

or

$$\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M}) \tag{8.33}$$

The relationship in eq. (8.33) holds for all materials whether they are linear or not. The concepts of linearity, isotropy, and homogeneity introduced in Section 5.7 for dielectric media equally apply here for magnetic media. For linear materials, $\mathbf{M}$ (in A/m) depends linearly on $\mathbf{H}$ such that

$$\mathbf{M} = \chi_m \mathbf{H} \tag{8.34}$$

where χ_m is a dimensionless quantity (ratio of M to H) called *magnetic susceptibility* of the medium. It is more or less a measure of how susceptible (or sensitive) the material is to a magnetic field. Substituting eq. (8.34) into eq. (8.33) yields

$$\mathbf{B} = \mu_o(1 + \chi_m)\mathbf{H} = \mu\mathbf{H} \tag{8.35}$$

or

$$\mathbf{B} = \mu_o\mu_r\mathbf{H} \tag{8.36}$$

where

$$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_o} \tag{8.37}$$

The quantity $\mu = \mu_o\mu_r$ is called the *permeability* of the material and is measured in henrys per meter; the henry is the unit of inductance and will be defined a little later. The dimensionless quantity μ_r is the ratio of the permeability of a given material to that of free space and is known as the *relative permeability* of the material.

It should be borne in mind that the relationships in eqs. (8.34) to (8.37) hold only for linear and isotropic materials. If the materials are anisotropic (e.g., crystals), eq. (8.33) still holds but eqs. (8.34) to (8.37) do not apply. In this case, μ has nine terms (similar to ε in eq. 5.37) and, consequently, the fields **B**, **H**, and **M** are no longer parallel.

†8.6 CLASSIFICATION OF MAGNETIC MATERIALS

In general, we may use the magnetic susceptibility χ_m or the relative permeability μ_r to classify materials in terms of their magnetic property or behavior. A material is said to be *nonmagnetic* if $\chi_m = 0$ (or $\mu_r = 1$); it is magnetic otherwise. Free space, air, and materials with $\chi_m = 0$ (or $\mu_r \approx 1$) are regarded as nonmagnetic.

Roughly speaking, magnetic materials may be grouped into three major classes: diamagnetic, paramagnetic, and ferromagnetic. This rough classification is depicted in Figure 8.13. A material is said to be *diamagnetic* if it has $\mu_r \lesssim 1$ (i.e., very small negative χ_m). It is *paramagnetic* if $\mu_r \gtrsim 1$ (i.e., very small positive χ_m). If $\mu_r \gg 1$ (i.e., very large positive χ_m), the material is *ferromagnetic*. Table B.3 in Appendix B presents the values μ_r for some materials. From the table, it is apparent that for most practical purposes we may assume that $\mu_r \simeq 1$ for diamagnetic and paramagnetic materials. Thus, we may regard diamagnetic and paramagnetic materials as linear and nonmagnetic. Ferromagnetic materials are always nonlinear and magnetic except when their temperatures are above curie temperature (to be explained later). The reason for this will become evident as we more closely examine each of these three types of magnetic material.

Diamagnetism occurs when the magnetic fields in a material that are due to electronic motions of orbiting and spinning completely cancel each other. Thus, the permanent (or intrinsic) magnetic moment of each atom is zero and such materials are weakly affected by a magnetic field. For most diamagnetic materials (e.g., bismuth, lead, copper, silicon, diamond, sodium chloride), χ_m is of the order of -10^{-5}. In certain materials, called *superconductors*,

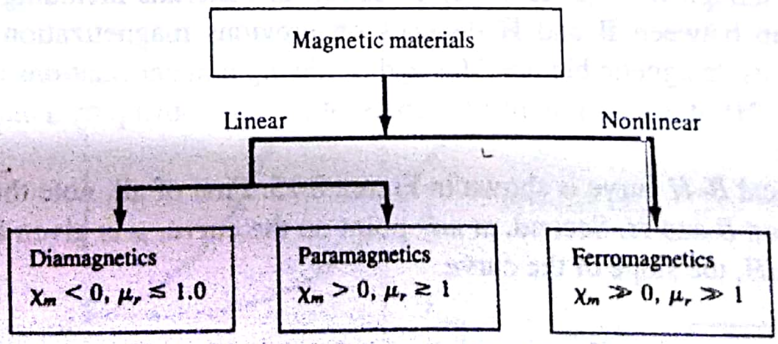

Figure 8.13 Classification of magnetic materials.

"perfect diamagnetism" occurs at temperatures near absolute zero: $\chi_m = -1$ or $\mu_r = 0$ and $B = 0$. Thus superconductors cannot contain magnetic fields.[2] Except for superconductors, the diamagnetic properties of materials are seldom used in practice. Although the diamagnetic effect is overshadowed by other stronger effects in some materials, all materials exhibit diamagnetism.

Materials whose atoms have nonzero permanent magnetic moment may be paramagnetic or ferromagnetic. *Paramagnetism* occurs when the magnetic fields produced in a material by orbital and spinning electrons do not cancel completely. Unlike diamagnetism, paramagnetism is temperature dependent. For most paramagnetic materials (e.g., air, platinum, tungsten, potassium), χ_m is of the order $+10^{-5}$ to $+10^{-3}$ and is temperature dependent. Such materials find application in masers.

Ferromagnetism occurs in materials whose atoms have relatively large permanent magnetic moment. They are called ferromagnetic materials because the best-known member is iron. Other members are cobalt, nickel, and their alloys. Ferromagnetic materials are very useful in practice. As distinct from diamagnetic and paramagnetic materials, ferromagnetic materials have the following properties:

1. They are capable of being magnetized very strongly by a magnetic field.
2. They retain a considerable amount of their magnetization when removed from the field.
3. They lose their ferromagnetic properties and become linear paramagnetic materials when the temperature is raised above a certain temperature known as the *curie temperature*. Thus if a permanent magnet is heated above its curie temperature (770°C for iron), it loses its magnetization completely.
4. They are nonlinear; that is, the constitutive relation $\mathbf{B} = \mu_o\mu_r\mathbf{H}$ does not hold for ferromagnetic materials because μ_r depends on $\mathbf{B}$ and cannot be represented by a single value.

Thus, the values of μ_r cited in Table B.3 for ferromagnetics are only typical. For example, for nickel $\mu_r = 50$ under some conditions and 600 under other conditions.

As mentioned in Section 5.9 for conductors, ferromagnetic materials, such as iron and steel, are used for screening (or shielding) to protect sensitive electrical devices from disturbances from strong magnetic fields. In the example of a typical iron shield shown in Figure 8.14(a), the compass is protected. Without the iron shield, as in Figure 8.14(b), the compass gives an erroneous reading owing to the effect of the external magnetic field. For perfect screening, it is required that the shield have infinite permeability.

Even though $\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$ holds for all materials including ferromagnetics, the relationship between $\mathbf{B}$ and $\mathbf{H}$ depends on previous magnetization of a ferromagnetic material—its "magnetic history." Instead of having a linear relationship between $\mathbf{B}$ and $\mathbf{H}$ (i.e., $\mathbf{B} = \mu\mathbf{H}$), it is only possible to represent the relationship by a *magnetization curve* or *B–H* curve.

A typical *B–H* curve is shown in Figure 8.15. First of all, note the nonlinear relationship between *B* and *H*. Second, at any point on the curve, μ is given by the ratio *B/H* and not by *dB/dH*, the slope of the curve.

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[2] An excellent treatment of superconductors is found in M. A. Plonus, *Applied Electromagnetics.* New York: McGraw-Hill, 1978, pp. 375–388. Also, the August 1989 issue of the *Proceedings of IEEE* is devoted to superconductivity.

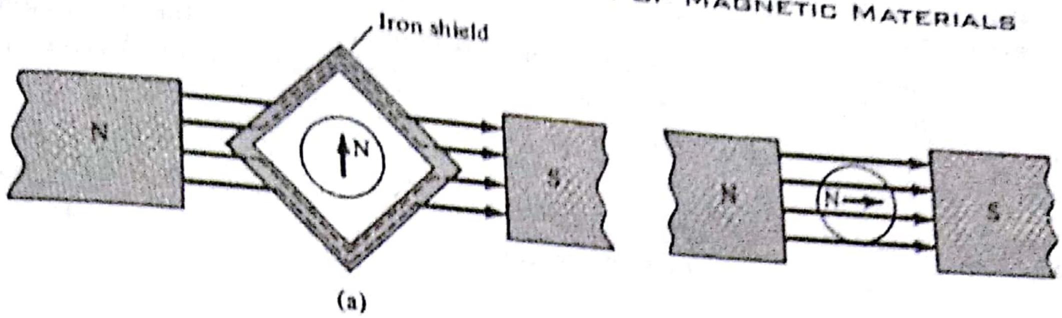

Figure 8.14 Magnetic screening: **(a)** iron shield protecting a small compass, **(b)** compass gives erroneous reading without the shield.

If we assume that the ferromagnetic material whose B–H curve in Figure 8.15 is initially unmagnetized, as H increases (owing to increase in current) from O to maximum applied field intensity H_{max}, curve OP is produced. This curve is referred to as the *virgin* or *initial magnetization curve.* After reaching saturation at P, if H is decreased, B does not follow the initial curve but lags behind H. This phenomenon of B lagging behind H is called *hysteresis* (which means "to lag" in Greek).

If H is reduced to zero, B is not reduced to zero but to B_r, which is referred to as the *permanent flux density.* The value of B_r depends on H_{max}, the maximum applied field intensity. The existence of B_r is the cause of having permanent magnets. If H increases negatively (by reversing the direction of current), B becomes zero when H becomes H_c, which is known as the *coercive field intensity.* Materials for which H_c is small are said to be magnetically hard. The value of H_c also depends on H_{max}.

Further increase in H in the negative direction to reach Q and a reverse in its direction to reach P gives a closed curve called a *hysteresis loop.* Hysteresis loops vary in shape from one material to another. Some ferrites, for example, have an almost rectangular hysteresis loop and are used in digital computers as magnetic information storage devices. The area of a hysteresis loop gives the energy loss (hysteresis loss) per unit volume during one cycle of the periodic magnetization of the ferromagnetic material. This

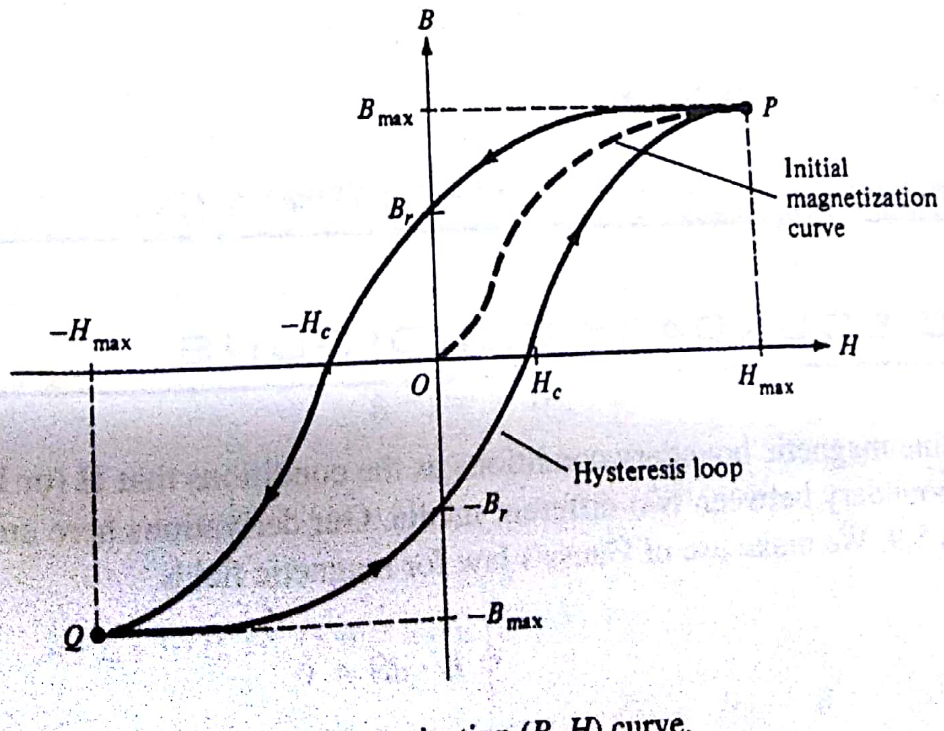

Figure 8.15 Typical magnetization (B–H) curve.

energy loss is in the form of heat. It is therefore desirable that materials used in electric generators, motors, and transformers have tall but narrow hysteresis loops so that hysteresis losses are minimal.

EXAMPLE 8.7

Region $0 \leq z \leq 2$ m is occupied by an infinite slab of permeable material ($\mu_r = 2.5$). If $\mathbf{B} = 10y\mathbf{a}_x - 5x\mathbf{a}_y$ mWb/m² within the slab, determine: (a) $\mathbf{J}$, (b) $\mathbf{J}_b$, (c) $\mathbf{M}$, (d) $\mathbf{K}_b$ on $z = 0$.

Solution:

(a) By definition,

$$\mathbf{J} = \nabla \times \mathbf{H} = \nabla \times \frac{\mathbf{B}}{\mu_o \mu_r} = \frac{1}{4\pi \times 10^{-7}(2.5)} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \mathbf{a}_z$$

$$= \frac{10^6}{\pi}(-5 - 10)10^{-3}\mathbf{a}_z = -4.775\mathbf{a}_z \text{ kA/m}^2$$

(b) $\mathbf{J}_b = \chi_m \mathbf{J} = (\mu_r - 1)\mathbf{J} = 1.5(-4.775\mathbf{a}_z) \cdot 10^3$
$= -7.163\mathbf{a}_z$ kA/m²

(c) $\mathbf{M} = \chi_m \mathbf{H} = \chi_m \dfrac{\mathbf{B}}{\mu_o \mu_r} = \dfrac{1.5(10y\mathbf{a}_x - 5x\mathbf{a}_y) \cdot 10^{-3}}{4\pi \times 10^{-7}(2.5)}$
$= 4.775y\mathbf{a}_x - 2.387x\mathbf{a}_y$ kA/m

(d) $\mathbf{K}_b = \mathbf{M} \times \mathbf{a}_n$. Since $z = 0$ is the lower side of the slab occupying $0 \leq z \leq 2$, $\mathbf{a}_n = -\mathbf{a}_z$. Hence,

$$\mathbf{K}_b = (4.775y\mathbf{a}_x - 2.387x\mathbf{a}_y) \times (-\mathbf{a}_z)$$
$$= 2.387x\mathbf{a}_x + 4.775y\mathbf{a}_y \text{ kA/m}$$

PRACTICE EXERCISE 8.7

In a certain region ($\mu = 4.6\mu_o$),

$$\mathbf{B} = 10e^{-y}\mathbf{a}_z \text{ mWb/m}^2$$

find: (a) χ_m, (b) $\mathbf{H}$, (c) $\mathbf{M}$.

Answer: (a) 3.6, (b) $1730e^{-y}\mathbf{a}_z$ A/m, (c) $6228e^{-y}\mathbf{a}_z$ A/m.