[This question paper contains 4 printed pages]

Your Roll No. : 17025567012

Sl. No. of Q. Paper : **2264** **IC**

Unique Paper Code : 32221401

Name of the Course : **B.Sc. (Hons.) Physics**

Name of the Paper : Mathematical Physics-III

Semester : IV

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **five** questions in all.

(c) Question **No.1** is compulsory.

(d) **All** questions carry equal marks.

(e) Attempt **two** questions from **Section - A** and **B** each.

1. Attempt any **five** questions : 5×3=15

(a) Find $(1 + i)^8$.

(b) Given a complex number z, represent geometrically iz in the Argand plane.

(c) Evaluate $\oint_C \frac{4-3z}{(z-1)(z-2)}dz$; if C is the circle $|z| = \frac{3}{2}$.

P.T.O.

(d) Locate and name all the singular points of the function (in finite z - plane) $\frac{\sin\sqrt{z}}{\sqrt{z}}$.

(e) If $F(\alpha)$ is the Fourier Transform of $f(x)$, then find the Fourier Transform of $f(x)\sin ax$, where, a is any positive number.

(f) Show that derivative of unit step function is a Dirac Delta function.

(g) If the Laplace Transform of f(t) is F(s), then show that the Laplace Transform of $\int_0^t F(u)du$ is $\frac{F(s)}{s}$.

(h) Find the Laplace Transform of function $f(t) = (\sin t - \cos t)^2$.

Section - A

2. (a) Construct an analytic function $f(z) = u + iv$, whose $u(x, y) = x^2 - y^2 - y$. 8

(b) Using De Moivre's theorem, verify that

$\cos 5\theta = 16\cos^2\theta - 20\cos^3\theta + 5\cos\theta$, and

$\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$ 7

3. (a) Expand $f(z) = \frac{z}{z-3}$ in a Laurent's series valid for the given regions : 8

(i) $|z| < 3$, (ii) $|z| > 3$

(b) Evaluate $\oint_C \frac{ze^{zt}}{(z+1)^3} dz$, where C is circle $|z| = 2$ and $t > 0$. 7

4. Using the method of contour integration prove any **two** of the following : 7.5×2=15

(a) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+9)(x^2+4)} dx = \frac{\pi}{5}$

(b) $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2} = \frac{2\pi p^2}{1-p^2}, 0 < p < 1$

(c) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

Section - B

5. (a) Find the Fourier Integral of the function : 8

$f(x) = e^{-kx}$, when $x > 0$, $k > 0$ and $f(-x) = f(x)$

Hence deduce that

$$\int_0^{\infty} \frac{\cos \pi u}{1+u^2} du = \frac{\pi}{2} e^{-x} \ ; \ x > 0$$

 P.T.O.

(b) Find the Fourier Transform of Gaussian function. 7

6. (a) Using Laplace Transforms, solve the following set of simultaneous differential equations :

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t$$

given that x(0) =1 and y(0) = 0. 8

(b) Find $L^{-1}\left\{\frac{1}{(s^2+a^2)(s^2+b^2)}\right\}; a^2 \neq b^2$ 7

7. (a) If f(t) = t^a and g(t) = t^b, where a and b are integers, then using the Convolution Theorem for Laplace Transform, prove that : 8

$$\int_0^1 y^a (1-y)^b \, dy = \frac{a!\, b!}{(a+b+1)!}$$

(b) Prove that the Fourier transform of an even function is even. 7