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This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: 6682	HC
Unique Paper Code	: 32221401	
Name of the Paper	: **Mathematical Physics - III**	
Name of the Course	: **B.Sc. (Hons.) Physics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks

Question no. 1 is compulsory

Attempt 2 questions from Section A and 2 questions from Section B.

Use of Scientific Calculators is allowed.

1. Attempt any **five** questions: (3x5 =15)

(a) Evaluate $(-27i)^{\frac{1}{3}}$

(b) Locate and name the singularities in the finite z plane of the function

$$f(z) = \frac{ln(z+3i)}{z^2}$$

(c) Evaluate $\oint_C \frac{z^2-z+1}{z-2}\, dz$ over a circle C in the positive sense. C is described by $|z| = \frac{1}{2}$.

(d) Test the analyticity of the function $f(z) = z^2$.

(e) Show that $\delta(ax) = \frac{\delta(x)}{|a|}$ where $\delta(x)$ is the Dirac Delta function and a is a constant.

(f) If $F(\omega)$ represents the Fourier transform of $f(t)$, then prove that the Fourier transform of $f(t) \cos at = \frac{1}{2}[\, F(\omega - a) + F(\omega + a)]$.

(g) Evaluate the Laplace transform of $f(t) = cos^2\, 2t$

(h) Determine the inverse Laplace transform of:

$$F(s) = \left\{\frac{e^{-2s}}{s^3}\right\}$$

SECTION A

Attempt any two questions from this Section.

2 (a) Given a function $v(x, y) = e^x \sin y$. Find the function $u(x, y)$ such that $f(z) = u + i\, v$ is analytic. Express $f(z)$ in terms of z.

(b) Prove that

$$1 + \cos 72° + \cos 144° + \cos 216° + \cos 288° = 0$$

using complex analysis.

(10, 5)

3. (a) Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$ and determine the region of convergence of this series.

(b) Find the value of the integral $\oint_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ over a circle C (in the positive sense) represented by $|z| = 1$.

(10, 5)

4. Using the method of contour integration prove any *two* of the following:

(a) $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$

(b) $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$

(c) $\int_0^\infty \frac{\cos mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}, m > 0$ $\left(7\frac{1}{2}, 7\frac{1}{2}\right)$

SECTION B

Attempt any two questions from this Section.

5 (a) Prove that the Fourier transform of a Gaussian function $\left(e^{-x^2}\right)$ is also a Gaussian function.

(b) Solve the one dimensional wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

for $t > 0, -\infty < x < \infty$.

$u(x,0) = f(x)$; $u_t(x,0) = 0$; where $u_t = \frac{\partial u}{\partial t}$ and v is the velocity of the wave. (5, 10)

6 (a) Verify the convolution theorem (Fourier transform) for

P. T. O.

$$f(t) = g(t) = \begin{cases} 1, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

(b) Given that $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{t \sin t}{2}$, determine $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$, where the symbol $\mathcal{L}^{-1}$ represents the inverse Laplace transform operator. (10, 5)

7. (a) A semi-infinite rod ($x > 0$) is initially at temperature zero. At time $t = 0$, a constant temperature $T_o > 0$ is applied and maintained at the face $x = 0$. Using Laplace transform, find the temperature at any point of the rod at any later time $t > 0$.

$$\left[\text{Given,} \quad L^{-1}\left(\frac{e^{-x\sqrt{s/k}}}{s}\right) = erfc\left(\frac{x}{2\sqrt{kt}}\right)\right]$$

(b) Using Laplace transform, prove that,

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} \, dt = \ln\frac{2}{3}$$

(10, 5)