This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

2017

S. No. of Question Paper : **2820**

Unique Paper Code : **32221401** **GC-4**

Name of the Paper : **Mathematical Physics-III**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks.

Question No. **1** is compulsory.

Attempt **2** questions from Section A

and **2** questions from Section B.

Use of scientific calculators is allowed.

1. Attempt any *five* questions : 5×3=15

(*a*) Determine the solutions of the equation $z^4 = 1$ where z is a complex number. Represent the solutions graphically.

P.T.O.

Scanned with CamScanner

(*b*) Locate the name the singularities in the finite z plane of the function :

$$f(z) = \frac{\ln(z-2)}{(z^2-1)^2}.$$

(*c*) Evaluate :

$$\oint_C \frac{3z^2-6}{z-2}\,dz$$

over a circle C in the counterclockwise direction. C is described by $|z| = \pi$.

(*d*) Find the real part of (i^{2i}).

(*e*) Show that $x\delta(x) = 0$, where $\delta(x)$ is the Dirac delta function.

(*f*) If $F(\omega)$ is the Fourier transform of $f(t)$, then prove that the Fourier transform of :

$$f(at) = \frac{1}{a}F\left(\frac{\omega}{a}\right)$$

where a is a constant.

(*g*) Determine the Laplace transform of :

$$f(t) = \sin^2 2t.$$

Scanned with CamScanner

(*h*) Evaluate :

$$\int_0^\infty t e^{-3t} \sin t \, dt$$

using Laplace transform.

Section A

Attempt any *two* questions from this section.

2. (*a*) Verify Cauchy's theorem for the function :

$$f(z) = 2z^2 + 3z - 7,$$

if C is a square with vertices at $-1 \pm i, 1 \pm i$.

(*b*) Using De Moivre's theorem, prove that : 10,5

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

3. (*a*) Expand :

$$f(z) = \frac{z}{(z-1)(2-z)}$$

in a Laurent's series valid for the given regions :

(*i*) $1 < |z| < 2$

(*ii*) $|z - 1| > 1$

Scanned with CamScanner

(*b*) Evaluate :

$$\frac{1}{2\pi i}\int_C \frac{\cos \pi z}{z^2 - 1}\, dz$$

around a square with vertices at $\pm i, 2 \pm i$. 10,5

4. Using the method of contour integration prove any two of the following : 7½+7½

(*a*) $\int_0^\infty \frac{dx}{1 + x^6} = \frac{\pi}{3}$

(*b*) $\int_0^\pi \frac{d\theta}{1 + \sin^2 \theta} = \frac{\pi}{\sqrt{2}}$

(*c*) $\int_0^\infty \frac{\cos 3x}{(1 + x^2)(x^2 + 4)}\, dx = \frac{\pi}{2}\left(\frac{e^{-3}}{3} - \frac{e^{-6}}{6}\right).$

Section B

Attempt any *two* questions from this section.

5. (*a*) Determine the Fourier transform of the function $f(t)$:

$$f(t) = 1 - t^2 \text{ for } |t| < 1$$
$$= 0 \text{ otherwise}$$

Hence evaluate :

$$\int_0^\infty \frac{t\cos t - \sin t}{t^3} \cos \frac{t}{2}\, dt$$

(*b*) State and prove the convolution theorem for Fourier transforms. 8,7

Scanned with CamScanner

6. (a) Given $f(t) = 1$ for $-1 < t < 1$;

$= 0$ otherwise

Express $f(t)$ as a Fourier integral and hence evaluate :

$$\int_0^\infty \frac{\sin \omega \cos \omega t}{\omega} d\omega .$$

(b) State the convolution theorem for Laplace transform. Use this theorem to evaluate the inverse Laplace transform of : 8,7

$$F(s) = \frac{1}{s^2 (s^2 + 1)}$$

7. (a) Using Laplace transform, solve the following set of simultaneous differential equations :

$$\frac{dx(t)}{dt} + y(t) = 0$$
$$\frac{dy(t)}{dt} - x(t) = 0 \text{ given } x(0) = 1; \, y(0) = 0 .$$

(b) Determine the Laplace transform of a periodic function $f(t)$ with period T. 10,5

Scanned with CamScanner