Weekly e-material (13/4/2020 to 17/4/2020) (1/22)

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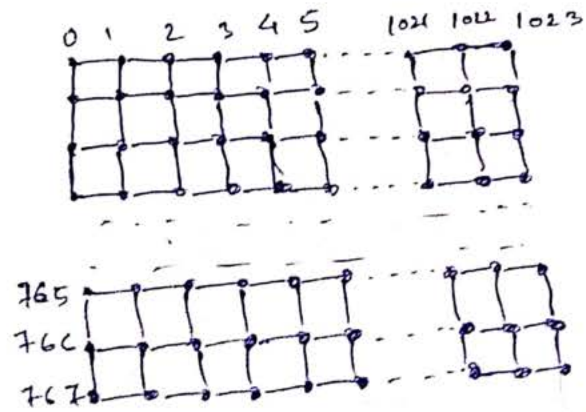
paper Title :- Linear Algebra
class :- Chemistry (H) 1st year.
Chapter 5 (Finite-Dimensional vector spaces)

Computer Graphics.

Computer graphics is an art of drawing pictures on Computer screens with the help of programming. It involves computations, creation and manipulation of data. A Computer screen consist of all Collection ~~and~~ of tiny, uniformly sized pixels (picture elements or dots), which are arranged in a two-dimensional grid made up of columns and rows, with a single pixel at the intersection of each rows and column.



The number of pixels, Called resolution, affects how much detail Can be ~~dip~~ depicted in an image. Resolution is often expressed as the number of pixels in a row times number of pixels in a column.

for example, a typical 1024 x 768 computer screen would have 1024 pixels in each row (labeled 0 through 1023) and 768 pixels in each column (labeled 0 through 767).

The most common computer graphics technique is raster graphics, in which the current screen content (text, figure, icons, etc.) is stored in the memory of the computer and updated and displayed whenever a change of screen contents is necessary.

In this system, a simple two-dimensional figure with n vertices can be represented by a 2xn matrix, with each column of the matrix lists of a pair of x-coordinate and y-coordinate representing a different vertex of the figure.

For example, Consider the polygon in figure 1 (a tree with 6 vertices. We can represent this polygon algebrically by storing it's 6 vertices as 6 columns in the following 2x6 matrix.

$$\begin{bmatrix} 8 & 8 & 6 & 8 & 10 & 10 \\ 2 & 8 & 10 & 12 & 10 & 6 \end{bmatrix}$$

~~Followed~~

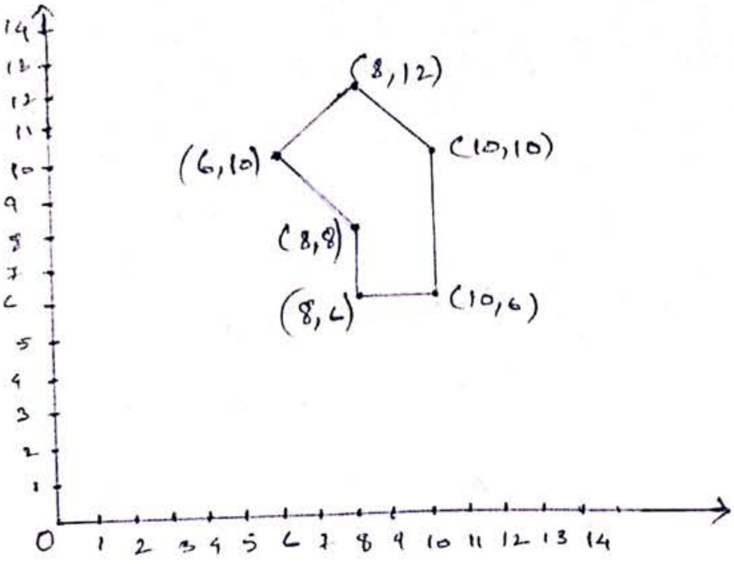


FIGURE 1

Wherever we move a given figure on the screen, the x-coordinate and y-coordinate of new vertices of the figure may not be integers. For simplicity, we assume that whenever a figure a figure is manipulated (by rotation, reflection, or scaling), we round off each x- and y-coordinates to the nearest integers.

Fundamental Movements in the plane.

A similarity is a mapping of the plane to itself that moves every figure in the plane to another figure so that the figure and its image are similar in shape and related by the same ratio of sizes. It can be shown that any similarity can be accomplished by composing one or more of the following mappings.

1) Translation : A translation is a mapping that shifts all points of a figure along a given vector.

2) Rotations : A rotation is a mapping that rotates all points of a figure about a fixed point (called the center of rotation), through a fixed angle θ. Thus, a rotation is a mapping that turns a figure around a fixed point.

Note :- Unless otherwise stated, all rotations are assumed to be in a counterclockwise direction in the plane.

3) Reflection : A reflection is a mapping that reflects all points of a figure about a given line. Thus, a reflection is a mapping that moves a figure by flipping it across the given line.

4) Scaling : A scaling is a mapping that changes the size of a figure by dilating (or contracting) the distance of all points in the figure from a center point.

NOTE :- Each of the first three fundamental movements does not change the shape or size of a figure. In other words, it maps a given figure to a congruent figure. Any such map is called an isometry.

We consider each movement briefly in turn. As we will see, all translations are straightforward, but we begin with only the simplest possible type of rotation (about the origin), reflection (about the origin), and scalar (with the origin as center point).

(1) Translation :- To perform a translation of a vertex along a vector $\begin{bmatrix} a \\ b \end{bmatrix}$, we simply add $\begin{bmatrix} a \\ b \end{bmatrix}$ to the vertex.

(2) Rotation about the origin :- To perform a rotation ~~reflection~~ of a vertex about the origin, we simply multiply on the left by the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(3) Reflection about a line through the origin :-

To perform a reflection of a vertex about the line $y = mx$ ~~origin~~, we simply multiply on the left by the matrix

$$\frac{1}{m^2+1}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$

Two special cases:-

(a) Line of reflection is the x-axis (i.e. $m=0$).

In this case, the reflection matrix is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(b) Line of reflection is the y-axis. In this case, the reflection matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

(4) Scaling from the origin: To perform scaling about the origin with scale factors of c in the x-direction and d in the y-direction, we multiply on the left by the matrix

$$\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

Example (1) For the graphic in Figure 2, use ordinary coordinates in $\mathbb{R}^2$ to find the new vertices after performing each indicated operation.

(a) translation along the vector $[4,-2]$.

(b) rotation about the origin through $\theta = 30^\circ$.

(c) reflection about the line $y = 3x$.

(d) scaling about the origin with scale factors of 4 in the x-direction and 2 in the y-direction.

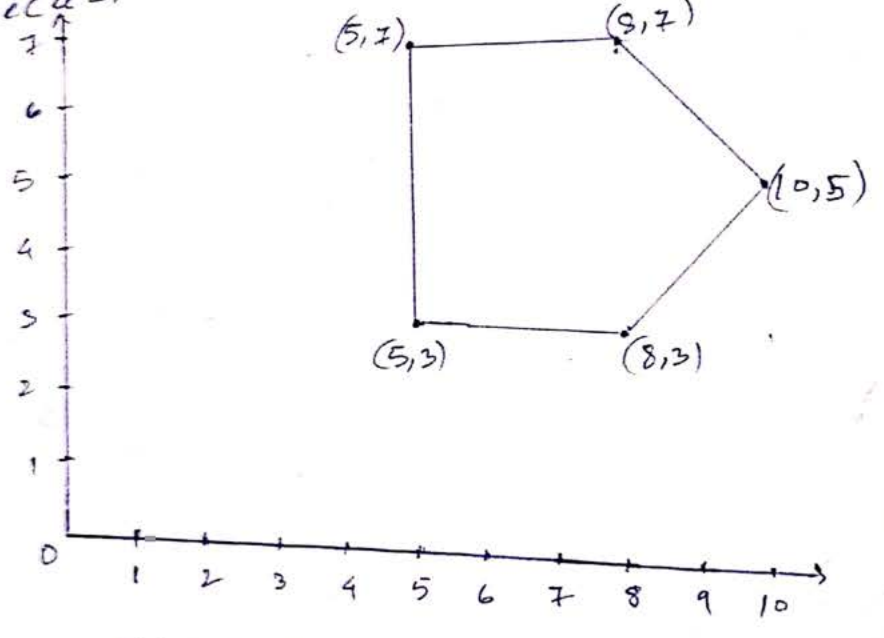


Figure 2

→ (a) To perform a translation of a vertex along the vector $[5,3]$, we simply add $\begin{bmatrix}4\\-2\end{bmatrix}$ to the vertex. Thus, adding $\begin{bmatrix}4\\-2\end{bmatrix}$ to each of the vertices of the given polygon, we obtain $\begin{bmatrix}5\\3\end{bmatrix} + \begin{bmatrix}4\\-2\end{bmatrix} = \begin{bmatrix}5+4\\3-2\end{bmatrix} = \begin{bmatrix}9\\1\end{bmatrix}$, and so on

Hence, new vertices of the polygon after performing translation are (9,5), (12,5), (14,3), (12,1).

(b) A rotation of (x,y) about the origin through the angle $\theta = 30°$ can be accomplished by multiplying $\begin{bmatrix} x \\ y \end{bmatrix}$ on the left by the matrix

$$\begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix}$$

∴ i.e., multiplying the vertex $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ on the left by the matrix $\begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix}$ yields

$$\begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.866 \times 5 - 0.5 \times 3 \\ 0.5 \times 5 + 3 \times 0.866 \end{bmatrix}$$

$$= \begin{bmatrix} 2.83 \\ 5.098 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ [Since we round off x- and y- coordinates to the nearest integer]

Similarly, we can find other new vertices.

Note that the new vertices can also be obtained directly by performing the rotation on all vertices of the figure simultaneously.

$$\begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \end{bmatrix}$$

$$\approx \begin{bmatrix} 3 & 1 & 3 & 6 & 5 \\ 5 & 9 & 10 & 9 & 7 \end{bmatrix}$$

(c) A reflection of (x,y) about the line $y=-3x$ can be obtained by multiplying $\begin{bmatrix} x \\ y \end{bmatrix}$ on the left by the matrix

$$\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix} = \frac{1}{1+3^2}\begin{bmatrix} 1-3^2 & 2.3 \\ 2.3 & 3^2-1 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} -8 & 6 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{4}{5} & 3/5 \\ 3/5 & 4/5 \end{bmatrix}$$

performing the reflection on all vertices of the polygon simultaneously, we obtain.

$$\begin{bmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \end{bmatrix} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 \\ 3 & 7 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 8 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -2.2 & +0.2 & -2.2 & -5 & -4.6 \\ 5.4 & 8.6 & 9.8 & 11 & 7.8 \end{bmatrix} \quad (131)$$

$$\approx \begin{bmatrix} -2 & 0 & -2 & -5 & -5 \\ 5 & 9 & 10 & 11 & 8 \end{bmatrix}$$

(d) A scaling of (x, y) about the origin by a factor of $c = 4$ in the x-direction and $d = 2$ in y-direction can be accomplished by multiplying $\begin{bmatrix} x \\ y \end{bmatrix}$ on the left by the matrix $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

performing the scaling on all vertices of the polygon simultaneously, we obtain

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 20 & 32 & 40 & 32 \\ 6 & 14 & 14 & 10 & 6 \end{bmatrix}$$

Homogeneous Coordinates.

In homogeneous coordinates, we add a third coordinate to a point. Instead of being represented by the two-dimensional point (x, y), each point is represented by a three-dimensional point $(x, y, 1)$. We define by

(12) any three-dimensional point of the form $(tx, ty, t) = t(x, y, 1)$, where $t \neq 0$, to be equivalent to the ordinary two-dimensional point (x, y). Thus, for each two-dimensional point (x, y), there is an infinite set of homogeneous coordinates (tx, ty, t), $t \neq 0$ in three-dimensional that are all equivalent to (x, y). For example, the points $(2,3,1)$, $(4,6,2)$ and $(6,9,3)$ are all equivalent to $(2,3)$. A point in homogeneous coordinates is said to be in normalized form if its last coordinate is 1. For example the homogeneous coordinate $(2,3,1)$ is said to be in normalized form. Notice that any point in homogeneous coordinates can be normalized simply by dividing all three coordinates of a triple by its last coordinates. Thus, each two-dimensional point has a unique set of normalized homogeneous coordinates, which is said to be <u>its standard form</u>. For example, the points $(15,-9,6)$, $(10,-6,4)$ and $(5/2,-3/2,1)$ are all homogeneous coordinates for the two-dimensional point $(5/2,-3/2)$.

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in which its standard form is $(5/2, -3/2, 1)$.

Representing movements with matrix multiplication in homogeneous coordinates.

Translation: To translate vertex (x,y) along a given vector $[a,b]$, we first replace (x,y) with its equivalent vector $[x,y,1]$ in homogeneous coordinates. Then, we ~~multiply~~ multiply on the left by the matrix

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

This gives
$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ 1 \end{bmatrix}$$

which is equivalent to the two-dimensional point $(x+a, y+b)$, as desired.

Rotation: To rotate vertex (x,y) about the origin through angle θ, we multiply its equivalent vector $[x,y,1]$ in homogeneous coordinates on the left by the matrix
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This gives

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ 1 \end{bmatrix},$$

which is equivalent to the two-dimensional point $(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$, as desired.

<u>Reflection</u>: A reflection of (x, y) about the line $y = mx$ can be accomplished by multiplying $[x, y, 1]$ on the left by the matrix

$$\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m & 0 \\ 2m & m^2-1 & \\ 0 & 0 & 1+m^2 \end{bmatrix}$$

In particular, a reflection about the y-axis can be accomplished by multiplying on the left by the matrix $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

<u>Scaling</u>: A scaling of (x, y) about the origin by a factor of c in the x-direction and d in the y-direction can be accomplished by multiplying $[x, y, 1]$ on the left by the matrix

$$\begin{bmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Movements not centered at the origin: Similarity Method.

We now determine the matrices for rotations, reflections, and scaling that are not centered about the origin using a method, Called the Similarly Method.

Similarity Method.

Step 1. Use a translation to move the figure so that the rotation, reflection or scaling to be performed is 'about the origin'.

This means for rotation or scaling about a point $(r, s) \neq (0,0)$, we first apply the translation that takes (r,s) to $(0,0)$, and for reflection about the line $y = mx+b$, we vertically translate the plane down b units so that the line of reflection goes through the origin.

Step 2. perform the desired rotation, reflection, or scaling "about the origin".

Step 3. Apply the reverse translation to get the altered figure back to the original figure.

Example (1) For the graphic in figure 3, use homogeneous coordinates to find the new vertices ~~at~~ after performing rotation about (8,9) through $\theta = 120°$.

→ To find new vertices, we first replace each (x,y) with its equivalent vector $[x,y,1]$ in homogeneous coordinates and then follow the three steps of the similarly method.

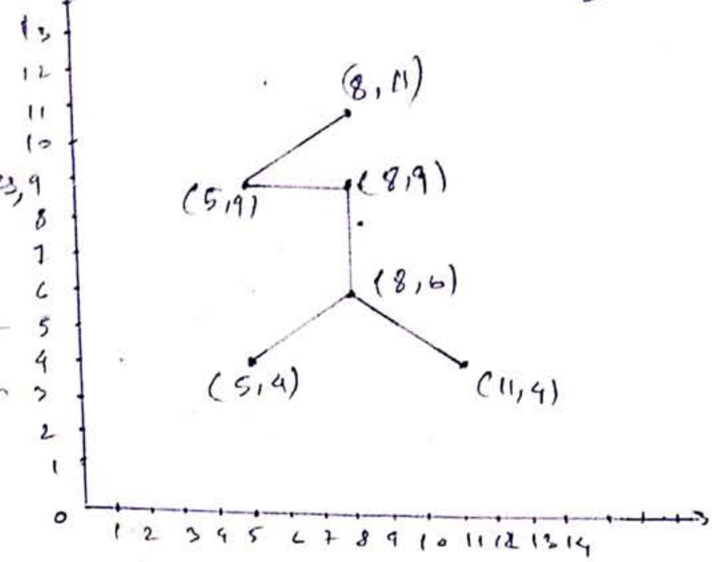

Figure 3.

step 1. We first apply the translation that takes (8,9) to (0,0) in order to establish the origin as center. The matrix for this operation is

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix}$$

step 2. The second step is to perform the rotation through angle $\theta = 120°$ about the origin. The matrix for this operation is

$$\begin{bmatrix} \cos 120° & -\sin 120° & 0 \\ \sin 120° & \cos 120° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(32)

$$= \begin{bmatrix} -0.5 & -0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3. Finally, we apply the reverse translation that takes (0,0) back to (8, 9). The matrix for

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}.$$

The combined result of these operations is

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & -0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & -0.866 & 19.794 \\ 0.866 & -0.5 & 6.572 \\ 0 & 0 & 1 \end{bmatrix}$$

performing the rotation on all vertices of the figure simultaneously, we obtain

$$\begin{bmatrix} -0.5 & -0.866 & 19.794 \\ 0.866 & -0.5 & 6.572 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 & 5 & 8 & 11 \\ 4 & 6 & 9 & 9 & 11 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 14 & 11 & 8 & 10 & 6 & 11 \\ 9 & 11 & 9 & 6 & 8 & 14 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(Q) For the graphic in Figure 4, use homogeneous coordinates to find the new vertices after performing reflection about the line $y = 4x - 10$. Then sketch the final figure that would result from this movement.

→ We first replace (x, y) with its equivalent vector $[x, y, 1]$, and follow the Similarity Method.

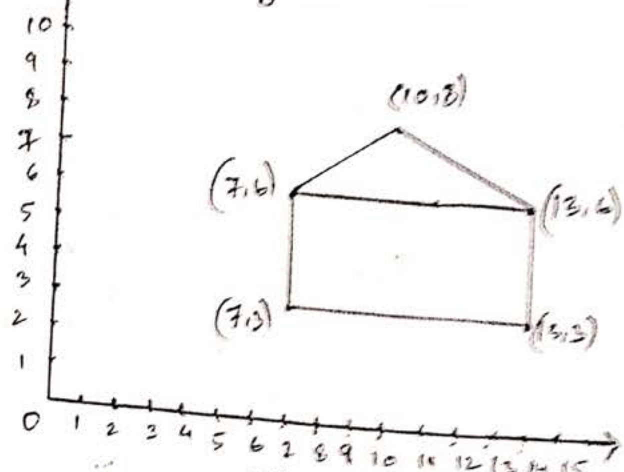

Figure 4

Step 1. The first step is to apply the translation so that the line $y = 4x - 10$ goes through the origin. This can be achieved by performing the translation that takes $(0, -10)$ to $(0, 0)$. The matrix for this operation is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2. We next perform a reflection about the corresponding line $y = 4x$. The matrix for this operation is

$$\frac{1}{1+4^2}\begin{bmatrix} 1-4^2 & 2\cdot4 & 0 \\ 2\cdot4 & 4^2-1 & 0 \\ 0 & 0 & 1+4^2 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -15 & 8 & 0 \\ 8 & 15 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

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Step 3. Finally, we apply the reverse translation that takes (0,0) back to (0, -10). The matrix for this operation is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

The combined result of these operations is

$$\frac{1}{17}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -15 & 8 & 0 \\ 8 & 15 & 0 \\ 0 & 0 & 17 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -15 & 8 & 0 \\ 8 & 15 & -170 \\ 0 & 0 & 17 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -15 & 8 & 80 \\ 8 & 15 & -20 \\ 0 & 0 & 17 \end{bmatrix}$$

performing the reflection on all vertices of the figure simultaneously, we obtain

$$\frac{1}{17}\begin{bmatrix} -15 & 8 & 80 \\ 8 & 15 & -20 \\ 0 & 0 & 17 \end{bmatrix}\begin{bmatrix} 7 & 7 & 10 & 13 & 13 \\ 3 & 6 & 8 & 3 & 6 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -1 & 23 & -6 & -91 & -67 \\ 81 & 126 & 180 & 129 & 174 \\ 17 & 17 & 17 & 17 & 17 \end{bmatrix}$$

$$\approx \begin{bmatrix} 0 & 1 & 0 & -5 & -4 \\ 5 & 7 & 11 & 8 & 10 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

after rounding the results for each vertex to the nearest integer.

The columns of the final matrix (ignoring the last entries) give the vertices of the reflected figure as shown in figure 5.

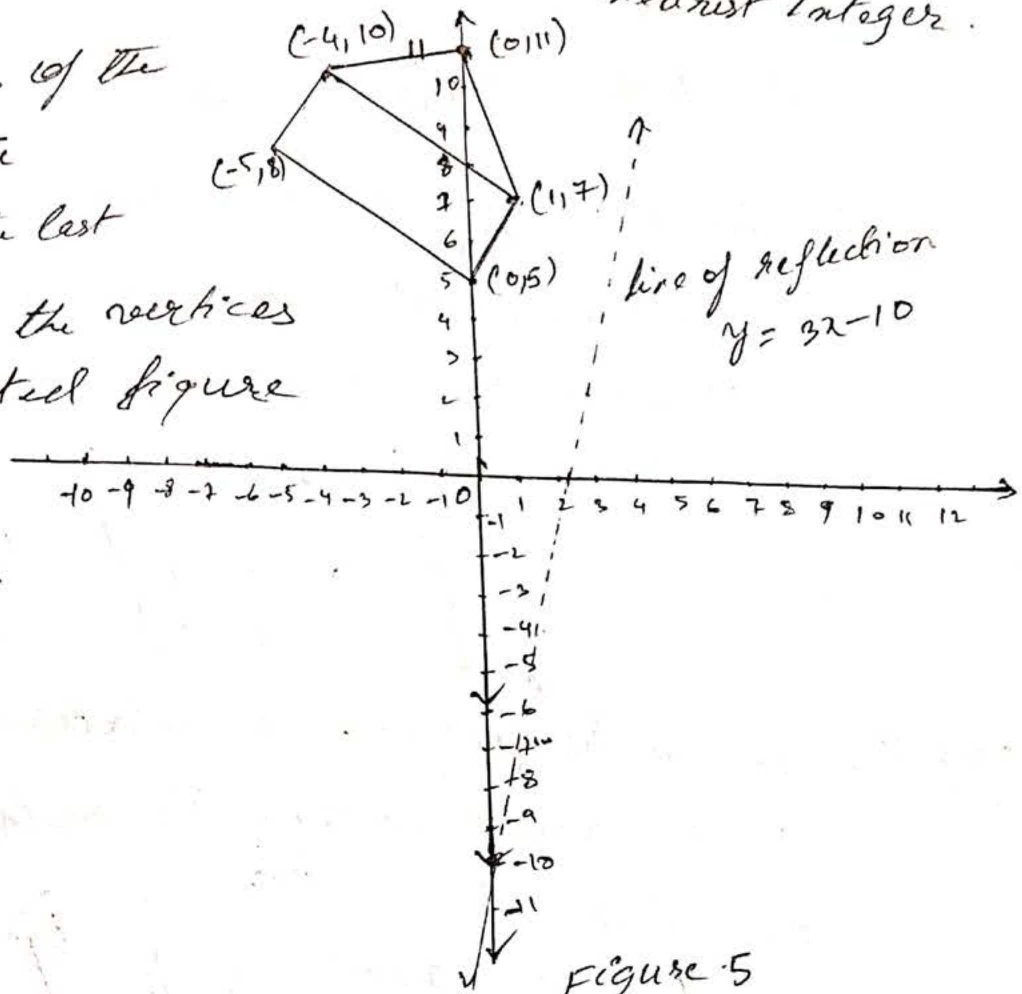

Figure 5

(3) (Composition of Movements)

Use the similarity method to show that a rotation about the point (1,-1) through an angle $\theta = 90°$, followed by a reflection about the line $x=1$ is represented by the matrix.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

. We first replace each (x, y) with its vector $[x, y, 1]$ in homogeneous coordinates and follow the Similarity Method.

step 1. We first apply the translation that takes $(1, -1)$ to $(0, 0)$ in order to establish the origin as center. The matrix of this operation is

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

step 2. We next perform a rotation through angle $\theta = 90°$ about origin. The matrix of this operation is

$$\begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

step 3. We then apply the reverse translation that takes $(0, 0)$ back to $(1, -1)$. The matrix of this operation is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, the net effect of these 3 operations is to rotate each vertex about $(1, -1)$ through an angle $\theta = 90°$. The combined result of these operations is:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Next, observe that the line $x=1$ is parallel to y-axis which passes through $(1,0)$. As before, we follow the similarity Method.

Step 1. We translate from $(1,0)$ to $(0,0)$ so that the line passes through the origin. The matrix of this operation is

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Step 2. We peform a reflection about the line $x=0$ (i.e. y-axis). The matrix of this operation is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3. We translate from $(0,0)$ back to $(1,0)$. The matrix of this operation is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The combined result of these operations is:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, the matrix for the given composition is the (143)
product of the following ~~three~~ two matrices

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$