Name of the Teacher-Chitra
Guest lecturer, Department of physics and Electronics
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Applied Physics

Lecture-7 & 8

Unit II-Mechanical Properties of solids

Topics to be discussed in this lecture:

- Theoretical strength of a crystal
  - ✓ Theoretical shear stress
  - ✓ Theoretical cleavage stress
- Critically resolved shear stress

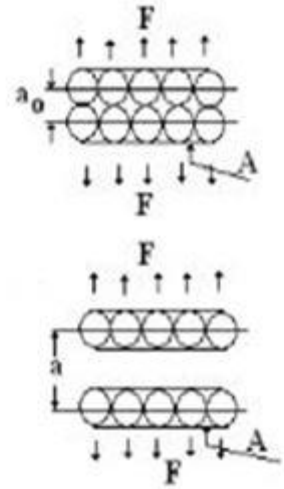
Theoretical Strength of a Crystal

- **The theoretical strength of a material with a perfect lattice is very high and of great importance.**
- **Theoretical strength is determined by the nature of :**
  - Inter-atomic forces
  - Temperature (causes atoms to vibrate)
  - Stress state of the material. We shall make calculations of two states of stresses: uniaxial normal stress and shear stress
- The stresses required for failure under the two situations will be calculated, and the theoretical strength should be the lower of the two values.

Theoretical Cleavage Stress

Materials, when exposed to external forces, will deform or fail. This may occur by

Cleavage:



- When a material breaks under perpendicular (i.e., normal) stress and the fracture is perpendicular to the applied stress.
- Consumes little energy and produces a brittle fracture.
- Cleavage involves the separation of atomic bonds perpendicular to an applied load/stress.

- **Orowan developed a simple method for obtaining the theoretical tensile strength of a crystal. It is assumed that all atoms separate simultaneously once their separation reaches a critical value. See Figure 1 below.**

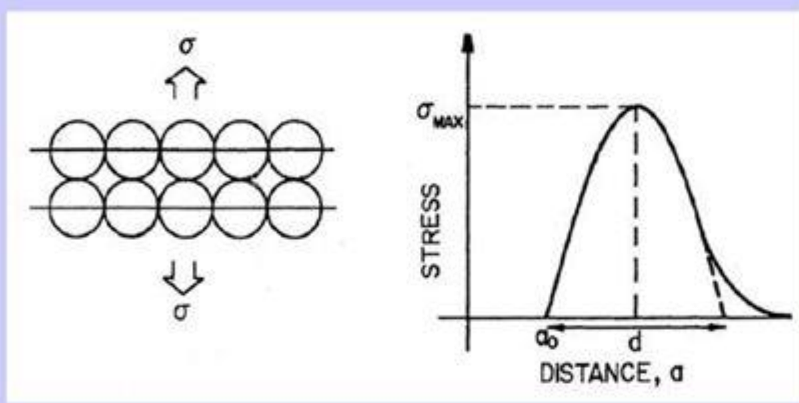


Figure -1. Stress required to separate two atomic layers. The stress will vary as a function of the distance between planes.

- The distance is initially equal to a_o.

- Naturally, $\sigma = 0$ for $a = a_o$; σ will also be zero when the separation is infinite. The exact form of the curve of σ versus a depends on the nature of the inter-atomic forces.

- In Orowan's model, the curve is simply assumed to be a sine function, hence there exists a generality of this model.

- The area under the curve is the work required to cleave the crystal.

- The stress dependence on plane separation is then given by the following equation:

$$\sigma = K \sin\frac{2\pi}{2d}(a-a_o) \tag{1}$$

- K can be determined from the differentiation of equation 1, and it is given as:

$$K = \frac{a.d}{\pi.a_o} \tag{2}$$

- Following a rigorous calculation, we obtain:

- $$\sigma_{max} = \sqrt{\frac{E.\gamma}{a_o}} \tag{3}$$

  where E is the Young's modulus and γ is the surface energy per unit volume.

- According to Orowan's model, the surface energy is given by:

- $$\gamma = \frac{K.d}{\pi} = \frac{E}{a_o}\left(\frac{d}{\pi}\right)^2 \tag{4}$$

- It has been experimentally determined that d is approximately equal to a_o. Hence,

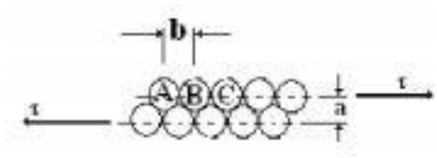
- $\gamma = \frac{E.a_o}{10}$ and $\sigma_{max} = \frac{E}{\pi}$ (5)

- We can conclude from Eq. 3 that, in order to have high theoretical cleavage strength, a material must have a high Young's modulus and surface energy and a small distance a_o between atomic planes.

Theoretical Shear Stress

Another kind of failure can be seen in crystals under application of external forces which is **Shear.**

Shear-



- When a material breaks under "parallel" stress and the fracture path is parallel to the applied stress.
- It consumes more energy as shear distorts specimen.
- Bond breakage is sequential.

- Frenkel performed a simple calculation of the theoretical shear strength of crystals by considering two adjacent and parallel lines of atoms subjected to a shear stress as shown in Figure 2 (below).

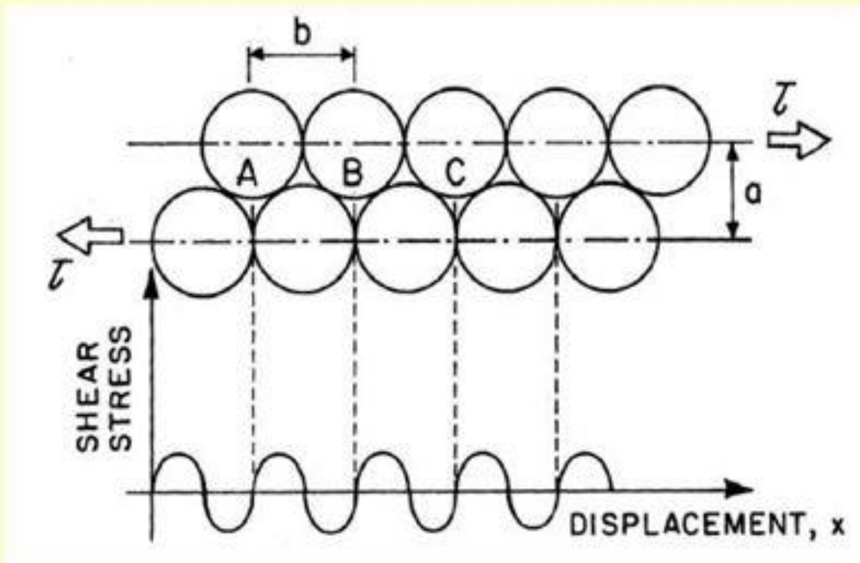


Figure -2. Stress required to shear a crystal.

- "b" is the inter-atomic distance or the distance between atoms in the slip directions

- "a" is the separation between adjacent planes or the spacing between adjacent lattice planes.

- Under the action of the stress τ, the top line will move in relation to the bottom line; the atoms will pass through successive equilibrium positions A, B, C, for which τ is zero.

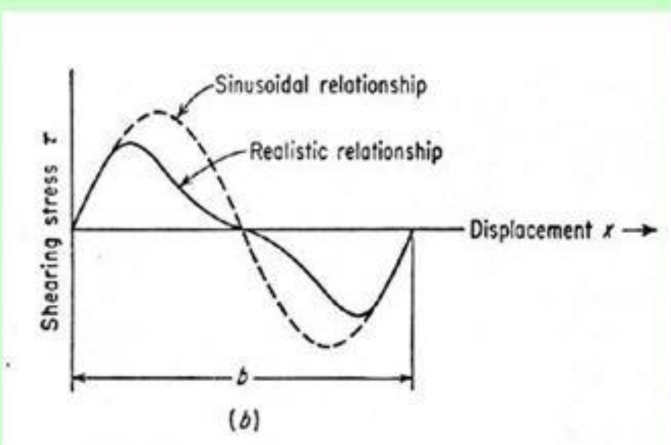


Figure 3. Variation of shearing stress with displacement in slip direction.

As a first approximation, the relationship between shear stress and displacement can be expressed by a sine function.

- $\tau = \tau_m \sin\frac{2\pi x}{b}$ (6)
- Where τ_m is the amplitude of the sine wave and b is the period. At small values of displacement, Hooke's law should apply.
- $\tau = G.\gamma = \frac{G.x}{a}$ (7)
- For small values of x/b, Eqn. (6) can be written
- $\tau = \tau_m.\frac{2\pi x}{b}$ (8)
- Combining Equations. (7) and (8) provides an expression for the maximum shear stress at which slip should occur.
- $\tau_m = \frac{G}{2\pi}\frac{b}{a}$ (9)
- As a rough approximation, b can be taken equal to a, with the result that the theoretical shear strength of a perfect crystal is approximately equal to the shear modulus divided by 2π.
- $\tau_m = \frac{G}{2\pi}$ (10)

The theoretical strength derived above is on the order of gigapascals; but, the actual strength of materials is orders of magnitude below that.

Experimental shear stress << theoretical shear stress. Dislocations prevent ductile materials from obtaining their theoretical shear stress.

- Since the theoretical shear strength of metal crystals is at least 100 times greater than the observed shear strength, it must be concluded that a mechanism other than bodily shearing of planes of atoms is responsible for slip.

  Also **σ experimental << σ theoretical**. Cracks prevents brittle materials from obtaining their theoretical cleavage stress.

  The equations of theoretical cleavage and shear strengths predict exceedingly high strengths (on the order of GPas), and few materials reach such strengths. If materials were perfect, those values could be reached. However, all materials contain imperfections, either by design or inadvertently produced during processing.

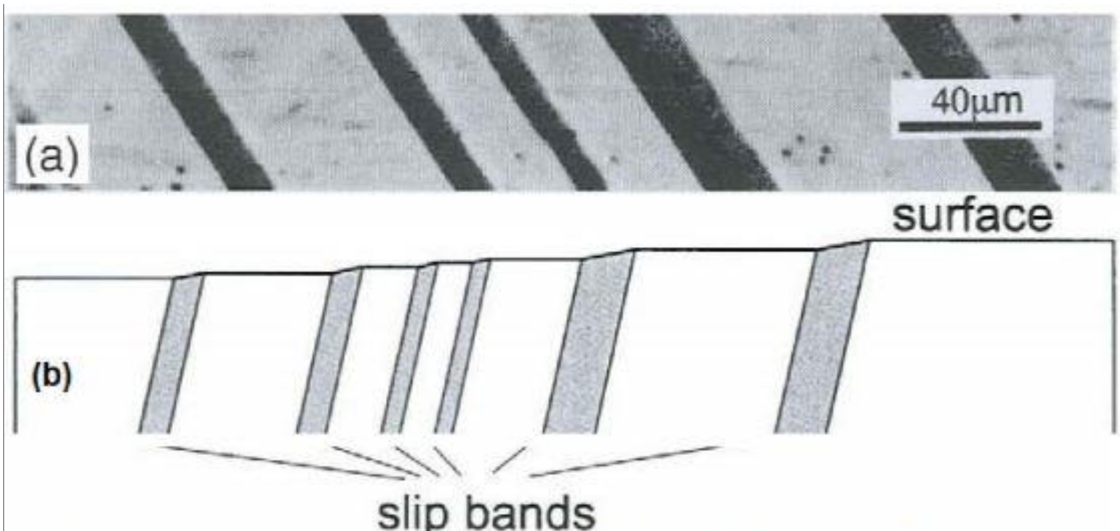
  These imperfections are mentioned below:

- **Point imperfection:** When the deviation from the periodic arrangement of the lattice is localized to the vicinity of only a few atoms it is called a point defect.

- **Lattice imperfection: If the defect extends through microscopic regions of the crystal. These may be divided into:**
  - line (one-dimensional) defects
  - surface or area (two-dimensional) defects
  - plane or volume (three-dimensional) defects

Critically resolved shear Stress

Slip results in the formation of steps on the surface of the crystal. These are readily detected if the surface is carefully polished before plastic deformation.



(a) Straight slip bands on a single crystal of 3.25 per cent silicon iron.
(b) Sketch of a section across the slip bands normal to surface shown in (a). Each band is made up of a large number of slip steps on closely spaced parallel slip planes.

Erich Schmid discovered that if a crystal is stressed, slip begins when the shear stress on a slip system reaches a critical value, $\boldsymbol{\tau_c}$, often called the critical resolved shear stress. In most crystals, slip occurs with equal ease forward or backward, so a characteristic shear stress is required for slip. Consider a crystal with a cross-sectional area A_0 being deformed in tension by an applied force F along the axis of the cylindrical crystal.

Plastic deformation of a single crystal is initiated at a critical stress, the critical resolved shear stress (CRSS). This is the same stress at which dislocations begin to move.

NORMAL Force: $F_N = F \cos\varphi$

SHEAR Force: $F_S = F \cos\lambda$.

• Area of slip-plane: $A_S = A_0 / \cos\varphi$

NORMAL Stress to the φ-plane: $\sigma = F_N / A_S$

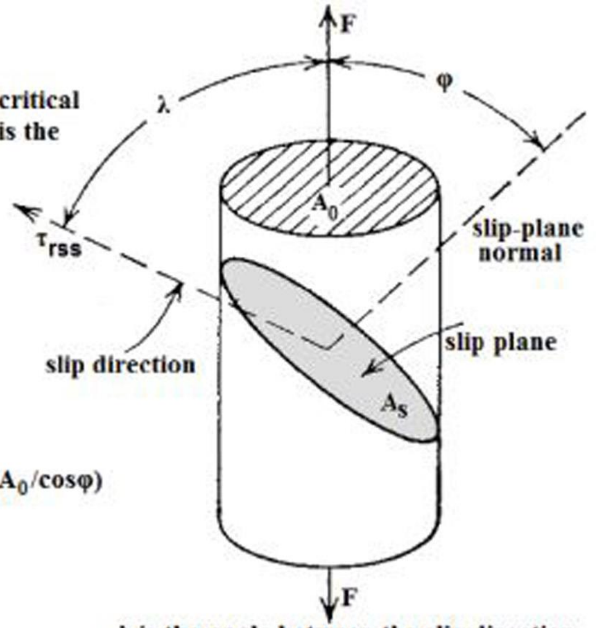
$$= (F\cos\varphi)/(A_0/\cos\varphi) = \sigma \cos^2\varphi$$

SHEAR Stress in the φ-plane: $\tau_S = F_S / A_S = (F\cos\lambda)/(A_0/\cos\varphi)$

$$= \sigma \cos\varphi \cos\lambda.$$

The shear stress causes slip to occur.

$$\boxed{\tau_{crss} = \sigma \cos\varphi \cos\lambda}$$



λ is the angle between the slip direction and the tensile axis
φ is the angle between the tensile axis and the slip-plane normal

where cosϕ cosλ is **Schmid Factor**. The active slip system will have the largest Schmid factor.

$\tau_{crss} = \tau_{max} == \frac{\sigma}{2}$ for $\lambda = \phi = 45^o$

$\tau_{crss} = 0$ for $\lambda = \phi = 90^o$ **(No slip, so fracture)**