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Applied Physics

Lecture-21 & 22

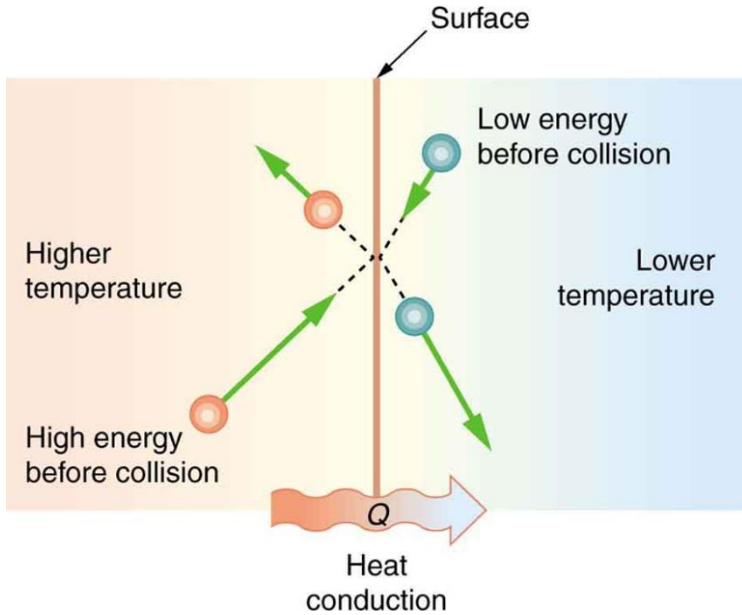
Unit III-Thermal Properties of solids

Topics to be discussed in this lecture:

- **Thermal Conduction**
- **Thermal conductivity of solids**
- **Thermal conductivity due to electrons**
- **Thermal conductivity due to phonons**

Thermal conduction:

In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. The figure below shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the hot to the cold molecule occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. We call this transfer of heat between two objects in contact **thermal conduction**.



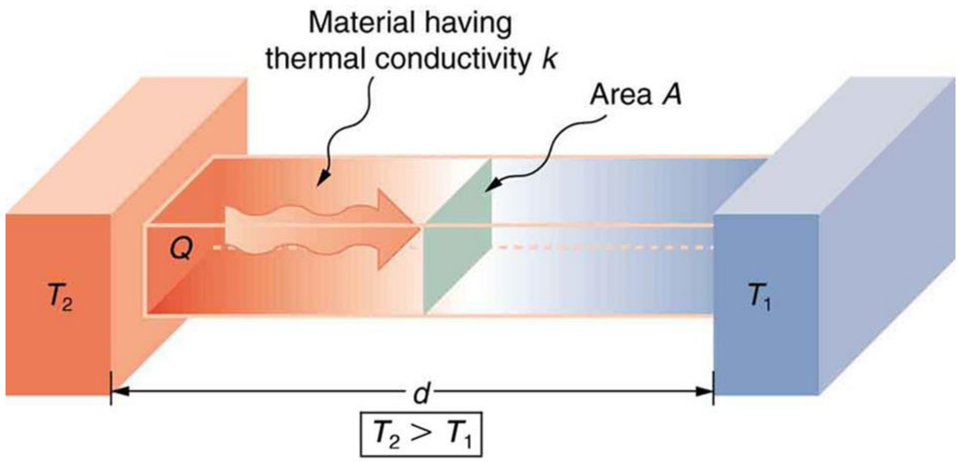
The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions.

What's the equation for the rate of thermal conduction?

There are four factors (k, A, ΔT, d) that affect the rate at which heat is conducted through a material. These four factors are included in the equation below that was deduced from and is confirmed by experiments.

$$\frac{Q}{t} = kA\frac{\Delta T}{d} \quad \textbf{(1)}$$

The letter Q represents the amount of heat transferred in a time t, k is the thermal conductivity constant for the material, A is the cross sectional area of the material transferring heat, ΔT is the difference in temperature between one side of the material and the other, and d is the thickness of the material. These factors can be seen visually in the diagram below.



Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber.

What does each term represent in the thermal conduction equation?

Let's look at what each factor means individually below.

***Q/t*:** The factor on the left hand side of the equation (1) represents the number of joules of heat energy transferred through the material per second. This means the quantity Q/t has unit joules/second=watts.

***k*:** The factor k is called the thermal conductivity constant. The thermal conductivity constant k is larger for materials that transfer heat well (like metal and stone), and k is small for materials that transfer heat poorly (like air and wood).

Δ*T*: The heat flow is proportional to the temperature difference $\Delta T = Thot - Tcold$ between one end of the conducting material and the other end. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved.

***A*:** Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area A. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

***d*:** A third factor in the mechanism of conduction is the thickness d of the material through which heat transfers. The figure above shows a slab of material with different temperatures on either side. Suppose that T_2 is greater T_1, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

Thermal conductivity of solids

Thermal conductivity is a process in which heat is transferred from one Part of the body to another as a result of temperature gradient. If the gradient is uniform then the amount of the thermal energy crossing a unit area in unit time is directly proportional to the temperature gradient, i.e.

$$Q \propto \frac{dT}{dx} \quad (2)$$

$$Q = K\frac{dT}{dx} \quad (3)$$

where the proportionality constant K is called the thermal conductivity In metallic conductors, heat is carried by free electrons, while in insulators, it is mainly carried by phonons. Therefore, in general the transfer of heat can take place both by electrons and phonons and hence the total thermal conductivity can be written as

$$K = K_{electron} + K_{phonon} \quad (4)$$

In order to determine the value of K in eq . 4, let us consider a conductor in the form of rod of uniform cross-section (say 1 sq.m) with the assumption that there exists a uniform temperature gradient. Let us draw three parallel planes at A , B and C normal to the direction of heat flow which are separated by one mean free path λ as shown in Fig .1.

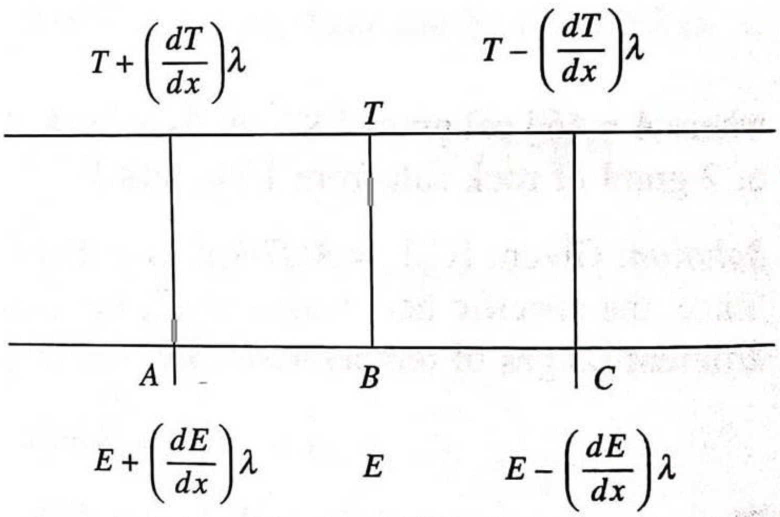


Fig. 1 Flow of heat through a conductor

According to the kinetic theory, the number of electrons flowing in a given direction through, unit area in unit time is $\frac{1}{6}(nv)$, where n is the density of electrons moving with an average velocity v . Further, the excess of energy carried by an electron from plane A to plane B is $(dE/dx)\lambda$. Thus the excess energy flowing through the plane B to the right is

$$Q_{right} = \frac{1}{6} n v \lambda \frac{dE}{dx} \qquad (5)$$

Similarly, the energy flowing through the plane B to the left is

$$Q_{left} = \frac{1}{6} n v \lambda \frac{dE}{dx} \qquad (7)$$

Therefore, the net amount of energy flowing through the plane B is

$$Q = Q_{right} - Q_{left} = \frac{1}{6} n v \lambda \frac{dE}{dx} - \left[-\frac{1}{6} n v \lambda \frac{dE}{dx}\right] = \frac{1}{3} n v \lambda \frac{dE}{dx} \qquad (8)$$

But $n\frac{dE}{dx}=n\frac{dE}{dT}\frac{dT}{dx}=nC_v\frac{dT}{dx}$

where C_v is the specific heal of solid at constant volume. Therefore, eq. 8 becomes

$$Q = \frac{1}{3}nC_v v\lambda\frac{dT}{dx} \tag{9}$$

Now, comparing eqs. 2 and 9 we obtain

$$K = \frac{1}{3}C_v v\lambda \tag{10}$$

Thermal conductivity due to Electrons

In order to derive an expression for thermal conductivity of metals where the heat is predominantly carried by the electrons, we make the following assumptions:

(i) The metal consists of fixed positive ions in a sea of electrons. In general, there will be one or two electrons per ion.

(ii) The electrons behave as a perfect gas and they transport thermal energy from the hotter to the colder region.

(iii) Each electron travels a distance λ in a mean free time τ before colliding with a positive ion where it gives up all its thermal energy.

(iv) Only those electrons that lie within the range kT of the Fermi level are active in the transport process. The velocity of such electrons is calculated from the formula

$$\frac{1}{2}mv_F^2 = E_F \tag{11}$$

Substituting the value of electronic specific heat as calculated from previous lecture (17 and 18) and λ = v_Fτ F r t in eq. 10 and making use of the eq. 11, we obtain the electronic contribution of thermal conductivity as

$$K_e = \left(\frac{2\tau E_F}{3m}\right)\left(\frac{\pi^2 N K^2}{2E_F}\right)T = \left(\frac{\pi^2 \tau N K^2}{3m}\right)T \qquad (12)$$

Since, the mean free time τ varies as T^{-1} above the Debye temperature, the electronic contribution of thermal conductivity is nearly temperature independent. This is consistent with the experimental facts. However, at low temperatures, the behavior is more complicated as illustrated in Fig. 2 for copper.

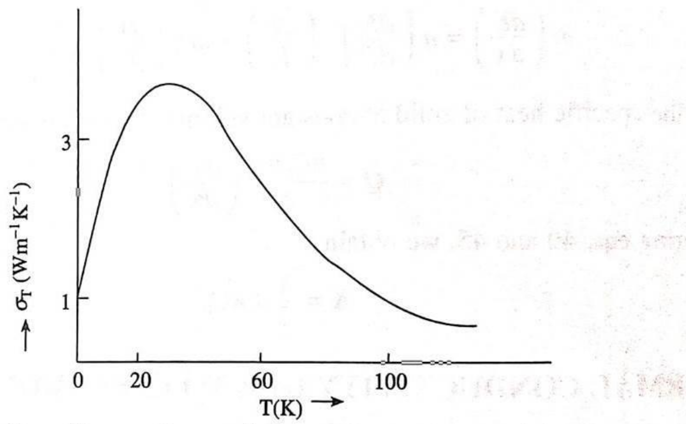


Fig. 2 Temperature variation of thermal conductivity for copper

Thermal conductivity due to phonons

As we know that in insulators, there are no mobile electrons and hence phonons carry most of the heat energy. Thus, for insulators in eq. 10, v is the velocity of sound, λ is the phonon mean free path and C_v is the lattice specific heat per unit volume given by the Debye T^3 law.

$$C_v = \alpha T^3 \qquad (T<\theta_D) \qquad (13)$$
$$= 3Nk \qquad (T>\theta_D)$$

Consequently the thermal conductivity of an insulator is proportional to T^3 at low temperatures as shown below in Fig. 3.

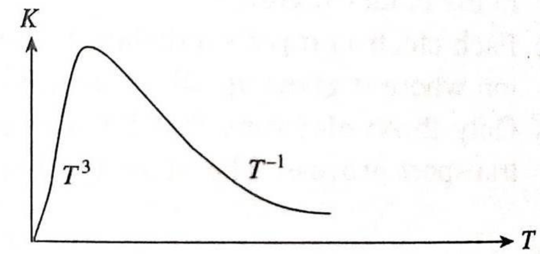


Fig. 3 Typical variation of phonon thermal conductivity with temperature

At high temperatures ($T \gg \theta_D$), the lattice specific heat is constant (i.e. equals to 3R, the Dulong Petit's law) and any temperature dependence of K arises predominantly from the variation of the mean free path λ due to phonon-phonon interaction (i.e. due to anharmonicity). However, a small contribution may also arise from collisions of phonons with crystal boundaries and impurities. Therefore, for all practical purposes only phonon-phonon contribution is considered. The lattice dynamics of anhannonic crystals which takes into account the phonon-phonon collision is very complicated, but leads to a simple final result, viz. the phonon mean free path is inversely proportional to the absolute temperature, i.e. $\lambda = T^{-1}$ so that ,

$$K = T^{-1} \tag{14}$$

Thus, the thermal conductivity of an insulator is proportional to T^{-1} at high temperatures.