Section 6.2 Numerical diff part II

① Forward diff approx :- for 1st derivatives

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi). \quad x_0 - h < \xi \le x_0$$

② Backward diff approx :-

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h} + \frac{h}{2} f''(\xi) \quad \text{replacing } x_0+h \text{ by } x_0-h.$$

③ F.D.A for 2nd order :-

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h} + \frac{h^2}{3} f'''(\xi).$$

④ BDA for 2nd order :-

$$f'(x_0) = \frac{3f(x_0) - 4f(x_0-h) + f(x_0-2h)}{2h} + \frac{h^2}{3} f'''(\xi)$$

replacing h by $-h$

⑤ Central Diff Approx for 1st

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6} f'''(\xi)$$

⑥ CDA for 2nd.

$$f''(x_0) = \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2} - \frac{h^2 f^{4}(\xi)}{12}.$$

Q16 (a) $f(x) = \ln x$, $x_0 = 2$.

$$f''(x_0) \approx \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$h = 1, 0.1, 0.01, 0.001$

ⓒ $f(x) = \ln x.$

$f'(x) = \frac{1}{x}.$

$f''(x) = -\frac{1}{x^2}.$

$f''(x_0) = -\frac{1}{4} = -0.25$

$h = 1.$

$\cancel{f''(2)}$ Let $\cancel{n} f(h) = \dfrac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}.$

$$f(1) = \frac{f(2-1) - 2f(2) + f(2+1)}{(1)^2}.$$

$$= \frac{\ln(1) - 2\ln(2) + \ln(3)}{1} = \frac{-2\ln(2) + \ln(3)}{1}.$$

$$= -0.287682072.$$

$$\approx f''(x_0).$$

$h = 0.1.$

$$f(0.1) = \frac{f(2-0.1) - 2f(2) + f(2+0.1)}{(0.1)^2}.$$

$$= \cancel{f} - 0.250031302$$

$$\approx f''(x_0).$$

$h = 0.01$

$$f(0.01) = \frac{f(2-0.01) - 2f(2) + f(2+0.01)}{(0.01)^2}$$

$$= -0.250003125$$

$$\approx f''(x_0).$$

$h = 0.001.$

$$f(0.001) = \frac{f(2-0.001) - 2f(2) + f(2+0.001)}{(0.001)^2}.$$

$$= -0.25000003.$$

$$\approx f''(x_0).$$

$$\therefore \quad f''(x_0) \approx \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}.$$

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Q11. (1) $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$. (T.S. this given f^n satisfy the given condn) while $f(x) = x^3$ doesn't satisfy.

(a) $$f'(x_0) \approx \frac{-3f(x_0) + 4.f(x_0+h) - f(x_0+2h)}{2h}.$$

~~LHS~~ ~~for~~ $f(x) = 1$.

$$f'(1) = \frac{-3(1) + \cancel{f(}4(1+h) - 1(1+2h)}{2h}$$

$$= \frac{-3+4+4h-1-2h}{2h}$$

$$= -4+4+2h$$

$$f'(1) \approx \frac{-3+4-1}{2h} = 0.$$

Again, $f(x) = 1$
$f'(x_0) = 0$.

(2). $f(x) = x$.
$f'(x) = 1$.

Again, $$f'(x_0) \approx \frac{-3x_0 + 4(x_0+h) - (x_0+2h)}{2h}$$

$$= \frac{-3x_0 + 4x_0 + 4h - x_0 - 2h}{2h}$$

$$= \frac{2h}{2h}$$

$$= 1.$$

$\therefore f'(x_0) = 1 = f'(x) = 1$.

(3) $f(x) = x^2$.
$f'(x) = 2x$.

$$f'(x_0) = \frac{-3x_0^2 + 4(x_0+h)^2 - (x_0+2h)^2}{2h}$$

$$f'(x_0) = \frac{-3x_0^2 + 4(x_0+h)^2 - (x_0+2h)^2}{2h}$$

$$= \frac{-3x_0^2 + 4x_0^2 + 8x_0h + 4h^2 - x_0^2 - 4x_0h - 4h^2}{2h}$$

$$= \frac{4x_0h}{2h}$$

$$= 2x_0$$

$\therefore \quad f'(x_0) = 2x_0 = f'(x) = 2x.$

(4) $f(x) = x^3$.

$f'(x) = 3x^2$.

$$f'(x_0) = \frac{-3x_0^3 + 4(x_0+h)^3 - (x_0+2h)^3}{2h}$$

$$= \frac{-3x_0^3 + 4(x_0^3 + 3x_0^2h + 3x_0h^2 + h^3) - x_0^3 - 6x_0^2h - 12x_0h^2 - 8h^3}{2h}$$

$$= \frac{-3x_0^3 + 4x_0^3 + 12x_0^2h + 12x_0h^2 + 4h^3 - x_0^3 - 6x_0^2h - 12x_0h^2 - 8h^3}{2h}$$

$$= \frac{6x_0^2h + 6x_0h^2 - 4h^3}{2h}$$

$$f'(x_0) = \frac{6x_0^2 + 6x_0 - 4h^2}{2} \neq f'(x) = 3x^2.$$

Q13 (a) $f'(x_0) \approx \dfrac{f(x_0+h) - f(x_0)}{h}$

$f(x) = 1 + x + x^3$. $\quad x_0 = 1, \quad h = 1, .1, .01, .001.$

$f'(x) = 1 + 3x^2$.

$f'(1) = 1 + 3$
$= 4$.

let $\quad f(h) = \dfrac{f(x_0+h) - f(x_0)}{h}$

$h = 1 \quad f(1) = \dfrac{f(1+1) - f(1)}{1} = \dfrac{(1+2+8) - (1+1+1)}{1}$

$= 11 - 3 = 9.$

$h = .1$ $f(.1) = \dfrac{f(1+.1) - f(0.1)}{(.1)}$

$= \dfrac{f(1.1) - f(0.1)}{(.1)} = \dfrac{3.431 - \cancel{1.000} \; 3}{(.1)} = \cancel{2.431} \; 4.31.$

$h = .01$ $f(0.01) = \dfrac{f(1.01) - f(\cancel{0.01}\;1)}{(0.01)} = \dfrac{3.040301 - \cancel{1.000000} \; 3}{}$

$= \cancel{2.040301} \; 4.0301.$

$h = 0.001.$ $f(0.001) = \dfrac{f(1.001) - f(\cancel{0.001}\;(1))}{(0.001)} = \dfrac{3.004003001 - \cancel{1.001000001}\;(3)}{(0.001)}.$

$= \cancel{2.003003001}.$

$= 4.003001.$

(b) $x_0 = 0.$

$f'(x_0) \approx \dfrac{f(x_0+h) - f(x_0)}{h}.$ $h = 1, .1, 0.01, 0.001.$

$f(x) = 1 + x + x^3.$

$f'(x) = 1 + 3x^2.$

$f'(0) = 1.$

$h = 1.$ $f'(1) = \dfrac{f(0+1) - f(0)}{1} = \dfrac{3-1}{1} = 2.$

$h = .1.$ $f'(.1) = \dfrac{f(.1) - f(0)}{(.1)} = 1.01.$

$h = 0.01.$ $f'(0.01) = \dfrac{f(0.01) - f(0)}{(0.01)} = 1.0001.$

$h = 0.001$ $f'(0.001) = \dfrac{f(0.001) - f(0)}{(0.001)} = 1.000001.$

$\therefore$ $f'(x_0) \approx \dfrac{f(x_0+h) - f(x_0)}{h}.$

i.e. $1 \approx 2.$ for $h = 1.$

$1 \approx 1.01.$ for $h = .1.$

$1 \approx 1.0001$ for $h = .01$

$1 \approx 1.000001$ for $h = .001.$