1. Introduction to Nuclear Physics

1.1 Basic Concepts

In this chapter we review some notations and basic concepts in Nuclear Physics. The chapter is meant to setup a common language for the rest of the material we will cover as well as rising questions that we will answer later on.

1.1.1 Terminology

A given atom is specified by the number of

- neutrons: N
- protons: Z
- electrons: there are Z electron in neutral atoms

Atoms of the same *element* have same atomic number Z. They are not all equal, however. *Isotopes* of the same element have different # of neutrons N .
Isotopes are denoted by ${}^A_Z X_N$ or more often by

$$\boxed{{}^A_Z X}$$

where X is the chemical symbol and $A = Z + N$ is the mass number. E.g.: ${}^{235}_{92}U, {}^{238}U$ [the Z number is redundant, thus it is often omitted].
When talking of different nuclei we can refer to them as

- Nuclide: atom/nucleus with a specific N and Z.
- Isobar: nuclides with same mass # A ($\neq Z, N$).
- Isotone: nuclides with same N, $\neq Z$.
- Isomer: same nuclide (but different energy state).

1.1.2 Units, dimensions and physical constants

Nuclear energies are measured in powers of the unit *Electronvolt*: 1eV $= 1.6 \times 10^{-19}$ J. The electronvolt corresponds to the kinetic energy gained by an electron accelerated through a potential difference of 1 volt. Nuclear energies are usually in the range of MeV (mega-electronvolt, or 10^6eV).
Nuclear masses are measured in terms of the *atomic mass unit*: 1 amu or 1u $= 1.66 \times 10^{-27}$kg. One amu is equivalent to 1/12 of the mass of a neutral ground-state atom of ^{12}C. Since electrons are much lighter than protons and neutrons (and protons and neutrons have similar mass), one nucleon has mass of about 1amu.
Because of the mass-energy equivalence, we will often express masses in terms of energy units. To convert between energy (in MeV) and mass (in amu) the conversion factor is of course the speed of light square (since $E = mc^2$). In these units we have: $c^2 = 931.502$ MeV/u.

- Proton mass: 938.280MeV/c^2.
- Neutron mass: 938.573MeV/c^2.
- Electron mass: 0.511MeV/c^2.

Note: you can find most of these values in Krane (and online!)
Scales of magnitude for typical lengths are femtometer (1fm=10^{-15}m) also called Fermi (F) and Angstrom $1\mathring{A} = 10^{-10}$m (for atomic properties) while typical time scales span a very broad range.
Physical constants that we will encounter include the speed of light, $c = 299,792,458$ m s^{-1}, the electron charge, $e = 1.602176487 \times 10^{-19}$ C, the Planck constant $h = 6.62606896 \times 10^{-34}$ J s and $\hbar$, Avogadro's number $N_a = 6.02214179 \times 10^{23}$ mol^{-1}, the permittivity of vacuum $\epsilon_0 = 8.854187817 \times 10^{-12}$ F m^{-1} (F=Faraday) and many others. A good reference (online) is NIST: http://physics.nist.gov/cuu/index.html
There you can also find a tool to convert energy in different units:
http://physics.nist.gov/cuu/Constants/energy.html

1.1.3 Nuclear Radius

The radius of a nucleus is not well defined, since we cannot describe a nucleus as a rigid sphere with a given radius. However, we can still have a practical definition for the range at which the density of the nucleons inside a nucleus approximate our simple model of a sphere for many experimental situations (e.g. in scattering experiments).
A simple formula that links the nucleus radius to the number of nucleons is the *empirical radius formula*:

$$\boxed{R = R_0 A^{1/3}}$$

1.2 Binding energy and Semi-empirical mass formula

1.2.1 Binding energy

Two important nuclear propert that we want to study are the nuclear binding energy and the mass of nuclides.
You could think that since we know the masses of the proton and the neutron, we could simply find the masses of all nuclides with the simple formula: $m_N \stackrel{?}{=} Zm_p + Nm_n$. However, it is seen experimentally that this is not the case.
From special relativity theory, we know that to each mass corresponds some energy, $E = mc^2$. Then if we just sum up the masses of all the constituents of a nucleus we would have how much energy they represent. The mass of a nucleus is also related to its intrinsic energy. It thus makes sense that this is not only the sum of its constituent energies, since we expect that some other energy is spent to keep the nucleus together. If the energy were equal, then it wouldn't be favorable to have bound nuclei, and all the nuclei would be unstable, constantly changing from their bound state to a sum of protons and neutrons.
The binding energy of a nucleus is then given by the difference in mass energy between the nucleus and its constituents. For a nucleus ${}^A_Z X_N$ the binding energy B is given by

$$B = \left[Zm_p + Nm_n - m_N({}^A X)\right] c^2$$

However, we want to express this quantity in terms of experimentally accessible quantities. Thus we write the nuclear mass in terms of the atomic mass, that we can measure, $m_N({}^A X)c^2 = [m_A({}^A X) - Zm_e]c^2 + B_e$, where $m_A({}^A X)$ is the *atomic* mass of the nucleus. We further neglect the electronic binding energy B_e by setting $m_N({}^A X)c^2 = [m_A({}^A X) - Zm_e]c^2$.

We finally obtain the expression for the nuclear binding energy :

$$B = \left\{ Zm_p + Nm_n - [m_A(^AX) - Zm_e] \right\} c^2$$

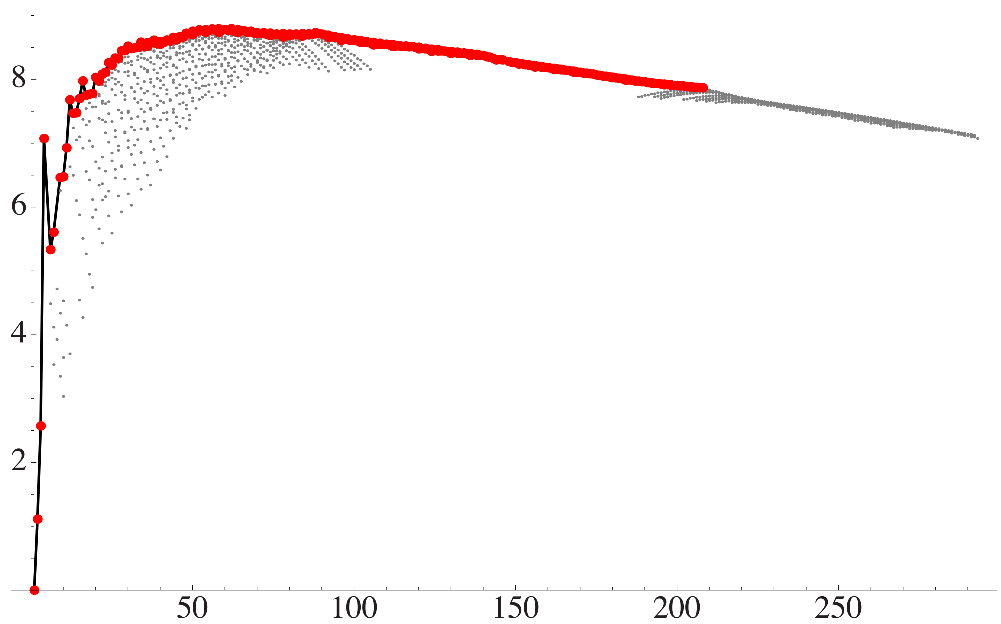

Fig. 1: Binding energy per nucleon (B/A in MeV vs. A) of stables nuclides (Red) and unstable nuclides (Gray).

Quantities of interest are also the neutron and proton separation energies:

$$S_n = B(^A_ZX_N) - B(^{A-1}_{Z}X_{N-1})$$

$$S_p = B(^A_ZX_N) - B(^{A-1}_{Z-1}X_N)$$

which are the analogous of the ionization energies in atomic physics, reflecting the energies of the *valence* nucleons. We will see that these energies show signatures of the shell structure of nuclei.

1.2.2 Semi-empirical mass formula

The binding energy is usually plotted as B/A or binding energy per nucleon. This illustrates that the binding energy is overall simply proportional to A, since B/A is mostly constant.

There are however corrections to this trend. The dependence of B/A on A (and Z) is captured by the *semi-empirical mass formula*. This formula is based on first principle considerations (a model for the nuclear force) and on experimental evidence to find the exact parameters defining it. In this model, the so-called **liquid-drop** model, all nucleons are uniformly distributed inside a nucleus and are bound together by the nuclear force while the Coulomb interaction causes repulsion among protons. Characteristics of the nuclear force (its short range) and of the Coulomb interaction explain part of the semi-empirical mass formula. However, other (smaller) corrections have been introduced to take into account variations in the binding energy that emerge because of its quantum-mechanical nature (and that give rise to the **nuclear shell model**).

The semi-empirical mass formula (SEMF) is

$$M(Z, A) = Zm(^1H) + Nm_n - B(Z, A)/c^2$$

where the binding energy $B(Z, A)$ is given by the following formula:

$$B(A, Z) = a_vA \;\; - a_sA^{2/3} - a_cZ(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta a_pA^{-3/4}$$

$$\underset{\text{volume}}{\nearrow} \quad \underset{\text{surface}}{\uparrow} \quad \underset{\text{Coulomb}}{\uparrow} \quad \underset{\text{symmetry}}{\uparrow} \quad \underset{\text{pairing}}{\nwarrow}$$

We will now study each term in the SEMF.

A. Volume term

The first term is the volume term $a_v A$ that describes how the binding energy is mostly proportional to A. Why is that so?
Remember that the binding energy is a measure of the interaction among nucleons. Since nucleons are closely packed in the nucleus and the nuclear force has a very short range, each nucleon ends up interacting only with a few neighbors. This means that independently of the total number of nucleons, each one of them contribute in the same way. Thus the force is not proportional to $A(A-1)/2 \sim A^2$ (the total # of nucleons one nucleon can interact with) but it's simply proportional to A. The constant of proportionality is a fitting parameter that is found experimentally to be $a_v = 15.5$MeV.
This value is smaller than the binding energy of the nucleons to their neighbors as determined by the strength of the nuclear (strong) interaction. It is found (and we will study more later) that the energy binding one nucleon to the other nucleons is on the order of 50 MeV. The total binding energy is instead the difference between the interaction of a nucleon to its neighbor and the kinetic energy of the nucleon itself. As for electrons in an atom, the nucleons are fermions, thus they cannot all be in the same state with zero kinetic energy, but they will fill up all the kinetic energy levels according to Pauli's exclusion principle. This model, which takes into account the nuclear binding energy and the kinetic energy due to the filling of shells, indeed gives an accurate estimate for a_v.

B. Surface term

The surface term, $-a_s A^{2/3}$, also based on the strong force, is a correction to the volume term. We explained the volume term as arising from the fact that each nucleon interacts with a constant number of nucleons, independent of A. While this is valid for nucleons deep within the nucleus, those nucleons on the surface of the nucleus have fewer nearest neighbors. This term is similar to surface forces that arise for example in droplets of liquids, a mechanism that creates surface tension in liquids.
Since the volume force is proportional to $B_V \propto A$, we expect a surface force to be $\sim (B_V)^{2/3}$ (since the surface $S \sim V^{2/3}$). Also the term must be subtracted from the volume term and we expect the coefficient a_s to have a similar order of magnitude as a_v. In fact $a_s = 13 - 18$MeV.

C. Coulomb term

The third term $-a_c Z(Z-1)A^{-1/3}$ derives from the Coulomb interaction among protons, and of course is proportional to Z. This term is subtracted from the volume term since the Coulomb repulsion makes a nucleus containing many protons less favorable (more energetic).
To motivate the form of the term and estimate the coefficient a_c, the nucleus is modeled as a uniformly charged sphere. The potential energy of such a charge distribution is

$$E = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Q^2}{R}$$

since from the uniform distribution inside the sphere we have the charge $q(r) = \frac{4}{3}\pi r^3 \rho = Q\left(\frac{r}{R}\right)^3$ and the potential energy is then:

$$E = \frac{1}{4\pi\epsilon_0}\int dq(\vec{r})\,\frac{q(\vec{r})}{|\vec{r}|} = \frac{1}{4\pi\epsilon_0}\int d^3\vec{r}\rho\frac{q(\vec{r})}{|\vec{r}|} = \frac{1}{4\pi\epsilon_0}\int_0^R dr\,4\pi r^2\rho\,\frac{q(r)}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\left(4\pi\int_0^R dr\,\frac{3Q}{4\pi R^3}r^2 Q\left(\frac{r}{R}\right)^3\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0}\int_0^R dr\,\frac{3Q^2 r^4}{R^6} = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Q^2}{R}$$

Using the empirical radius formula $R = R_0 A^{1/3}$ and the total charge $Q^2 = e^2 Z(Z-1)$ (reflecting the fact that this term will appear only if $Z > 1$, i.e. if there are at least two protons) we have :

$$\frac{Q^2}{R} = \frac{e^2 Z(Z-1)}{R_0 A^{1/3}}$$

which gives the shape of the Coulomb term. Then the constant a_c can be estimated from $a_c \approx \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R_0}$, with $R_0 = 1.25$fm, to be $a_c \approx 0.691$ MeV, not far from the experimental value.

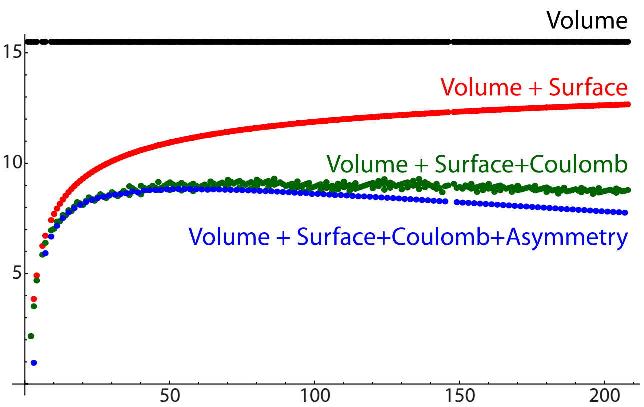

Fig. 2: SEMF for stable nuclides. We plot $B(Z,A)/A$ vs. A. The various term contributions are added one by one to arrive at the final formula.

D. Symmetry term

The Coulomb term seems to indicated that it would be favorable to have less protons in a nucleus and more neutrons. However, this is not the case and we have to invoke something beyond the liquid-drop model in order to explain the fact that we have roughly the same number of neutrons and protons in stable nuclei. There is thus a correction term in the SEMF which tries to take into account the symmetry in protons and neutrons. This correction (and the following one) can only be explained by a more complex model of the nucleus, the **shell model**, together with the quantum-mechanical *exclusion principle*, that we will study later in the class. If we were to add more neutrons, they will have to be more energetic, thus increasing the total energy of the nucleus. This increase more than off-set the Coulomb repulsion, so that it is more favorable to have an approximately equal number of protons and neutrons.

The shape of the symmetry term is $\frac{(A-2Z)^2}{A}$. It can be more easily understood by considering the fact that this term goes to zero for $A = 2Z$ and its effect is smaller for larger A (while for smaller nuclei the symmetry effect is more important). The coefficient is $a_{sym} = 23\text{MeV}$

E. Pairing term

The final term is linked to the physical evidence that like-nucleons tend to pair off. Then it means that the binding energy is greater ($\delta > 0$) if we have an even-even nucleus, where all the neutrons and all the protons are paired-off. If we have a nucleus with both an odd number of neutrons and of protons, it is thus favorable to convert one of the protons into a neutrons or vice-versa (of course, taking into account the other constraints above). Thus, with all other factor constant, we have to subtract ($\delta < 0$) a term from the binding energy for odd-odd configurations. Finally, for even-odd configurations we do not expect any influence from this pairing energy ($\delta = 0$). The pairing term is then

$$+\delta a_p A^{-3/4} = \begin{cases} +a_p A^{-3/4} \text{ even-even} \\ 0 \text{ even-odd} \\ -a_p A^{-3/4} \text{ odd-odd} \end{cases}$$

with $a_p \approx 34\text{MeV}$. [Sometimes the form $\propto A^{-1/2}$ is also found].

1.2.3 Line of Stability in the Chart of nuclides

By taking the first derivative wrt Z we can calculate the optimal Z such that the mass is minimum. We obtain:

$$Z_{min} = \frac{A}{2}\left(\frac{1+\frac{1}{4}A^{-1/3}\frac{a_c}{a_{sym}}}{1+\frac{1}{4}A^{2/3}\frac{a_c}{a_{sym}}}\right)$$

$$\approx \frac{A}{2}\left(1+\frac{1}{4}A^{2/3}\frac{a_c}{a_{sym}}\right)^{-1} \approx \frac{A}{2}\left(1-\frac{1}{4}A^{2/3}\frac{a_c}{a_{sym}}\right)$$

which gives $Z \approx \frac{A}{2}$ at small A, but has a correction for larger A such that $Z \approx 0.41A$ for heavy nuclei. [Note the approximation and series expansion is taken because $a_c \ll a_{sym}$]

If we plot Z/A vs. A the nuclides lie between 1/2 and 0.41. There is a line of stability, following the stable isotopes (red in figure 4 and black in figure 3). The isotopes are then variously labeled, for example here by their lifetime.

Interactive information is available at http://www.nndc.bnl.gov/chart/.

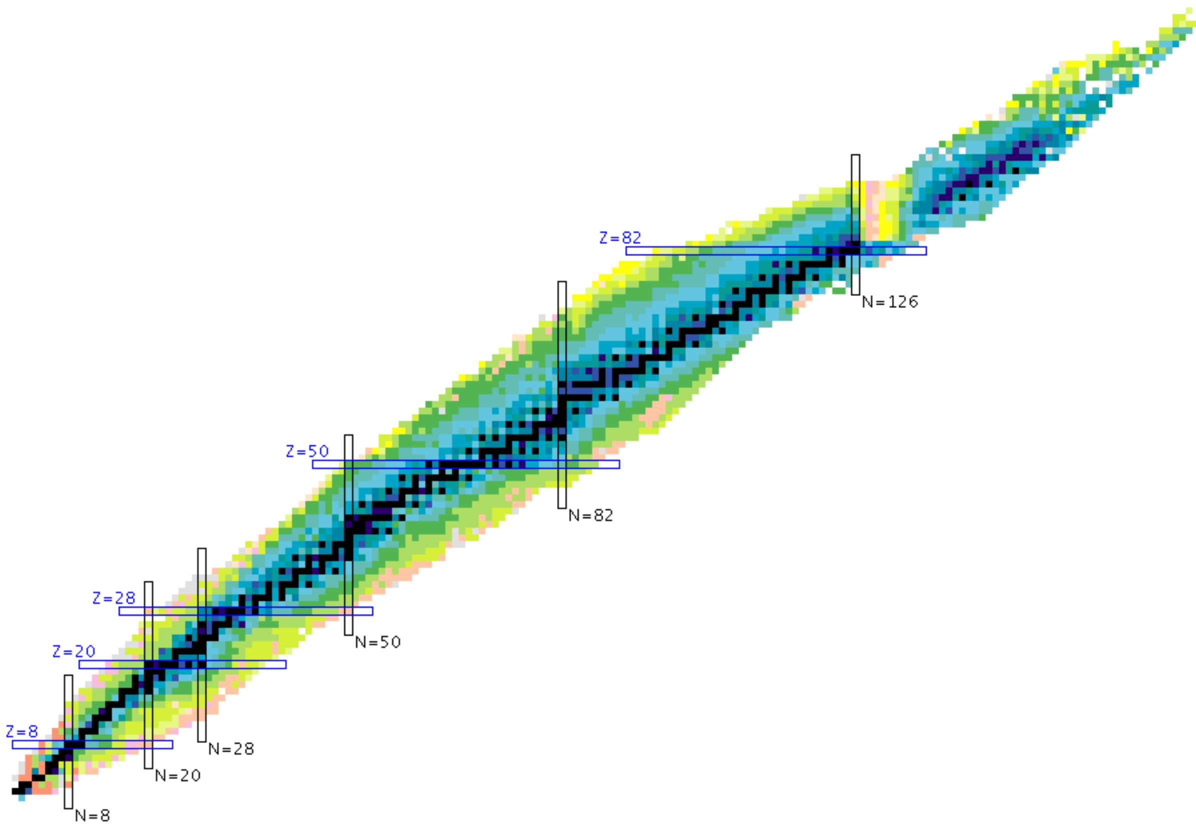

Fig. 3: Chart of nuclides from http://www.nndc.bnl.gov/chart/ . Each nuclide is color-labeled by its half-life (black for stable nuclides)

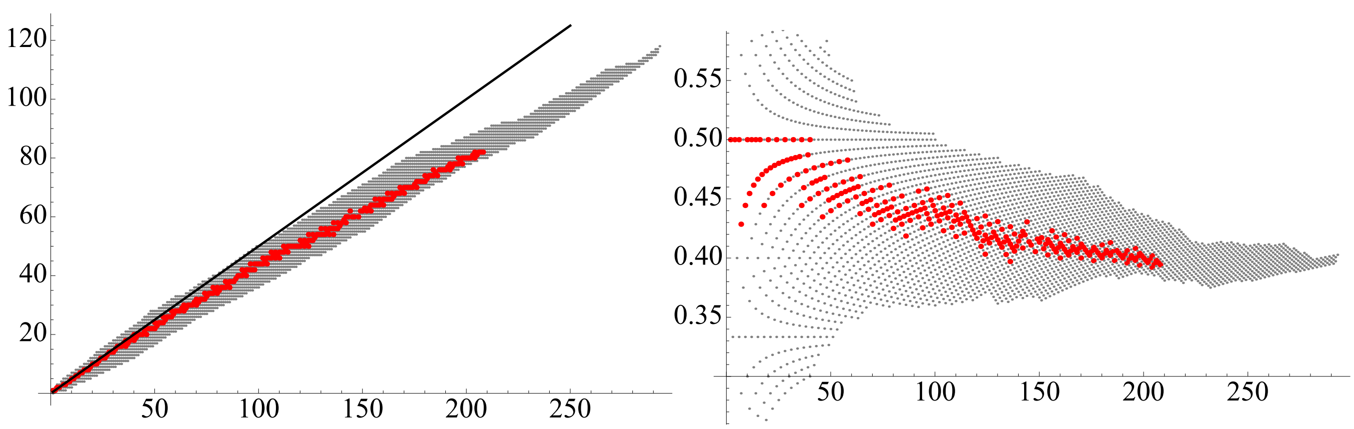

Fig. 4: Nuclide chart (obtained with the software Mathematica). Left: Z vs. A, Right: Z/A vs. A. In red, stable nuclides. The black line represents $Z = A/2$.

1.3 Radioactive decay

Radioactive decay is the process in which an unstable nucleus spontaneously loses energy by emitting ionizing particles and radiation. This decay, or loss of energy, results in an atom of one type, called the **parent** nuclide, transforming to an atom of a different type, named the **daughter** nuclide.
The three principal modes of decay are called the alpha, beta and gamma decays. We will study their differences and exact mechanisms later in the class. However these decay modes share some common feature that we describe now. What these radioactive decays describe are fundamentally quantum processes, i.e. transitions among two quantum states. Thus, the radioactive decay is statistical in nature, and we can only describe the evolution of the expectation values of quantities of interest, for example the number of atoms that decay per unit time. If we observe a single unstable nucleus, we cannot know a priori when it will decay to its daughter nuclide. The time at which the decay happens is random, thus at each instant we can have the parent nuclide with some probability p and the daughter with probability $1 - p$. This stochastic process can only be described in terms of the quantum mechanical evolution of the nucleus. However, if we look at an ensemble of nuclei, we can predict at each instant the average number of parent an daughter nuclides.
If we call the number of radioactive nuclei N, the number of decaying atoms per unit time is dN/dt. It is found that this rate is constant in time and it is proportional to the number of nuclei themselves:

$$\boxed{\frac{dN}{dt} = -\lambda N(t)}$$

The constant of proportionality λ is called the **decay constant**. We can also rewrite the above equation as

$$\lambda = -\frac{dN/dt}{N}$$

where the RHS is the probability per unit time for one atom to decay. The fact that this probability is a constant is a characteristic of all radioactive decay. It also leads to the *exponential law of radioactive decay*:

$$N(t) = N(0)e^{-\lambda t}$$

We can also define the **mean lifetime**

$$\boxed{\tau = 1/\lambda}$$

and the **half-life**

$$\boxed{t_{1/2} = \ln{(2)}/\lambda}$$

which is the time it takes for half of the atoms to decay, and the **activity**

$$\mathcal{A}(t) = \lambda N(t)$$

Since $\mathcal{A}$ can also be obtained as $|\frac{dN}{dt}|$, the activity can be estimated from the number of decays ΔN during a small time δt such that $\delta t \ll t_{1/2}$.
A common situation occurs when the daughter nuclide is also radioactive. Then we have a chain of radioactive decays, each governed by their decay laws. For example, in a chain $N_1 \rightarrow N_2 \rightarrow N_3$, the decay of N_1 and N_2 is given by:

$$dN_1 = -\lambda_1 N_1 dt, \qquad dN_2 = +\lambda_1 N_1 dt - \lambda_2 N_2 dt$$

Another common characteristic of radioactive decays is that they are a way for unstable nuclei to reach a more energetically favorable (hence stable) configuration. In α and β decays, a nucleus emits a α or β particle, trying to approach the most stable nuclide, while in the γ decay an excited state decays toward the ground state without changing nuclear species.

1.3.1 Alpha decay

If we go back to the binding energy per mass number plot (B/A vs. A) we see that there is a bump (a peak) for $A \sim 60 - 100$. This means that there is a corresponding minimum (or energy optimum) around these numbers. Then the heavier nuclei will want to decay toward this lighter nuclides, by shedding some protons and neutrons. More specifically, the decrease in binding energy at high A is due to Coulomb repulsion. Coulomb repulsion grows in fact as Z^2, much faster than the nuclear force which is $\propto A$.

This could be thought as a similar process to what happens in the fission process: from a parent nuclide, two daughter nuclides are created. In the α decay we have specifically:

$$^A_Z X_N \longrightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha$$

where α is the nucleus of He-4: $^4_2\text{He}_2$.
The α decay should be competing with other processes, such as the fission into equal daughter nuclides, or into pairs including ^{12}C or ^{16}O that have larger B/A then α. However α decay is usually favored. In order to understand this, we start by looking at the energetic of the decay, but we will need to study the quantum origin of the decay to arrive at a full explanation.

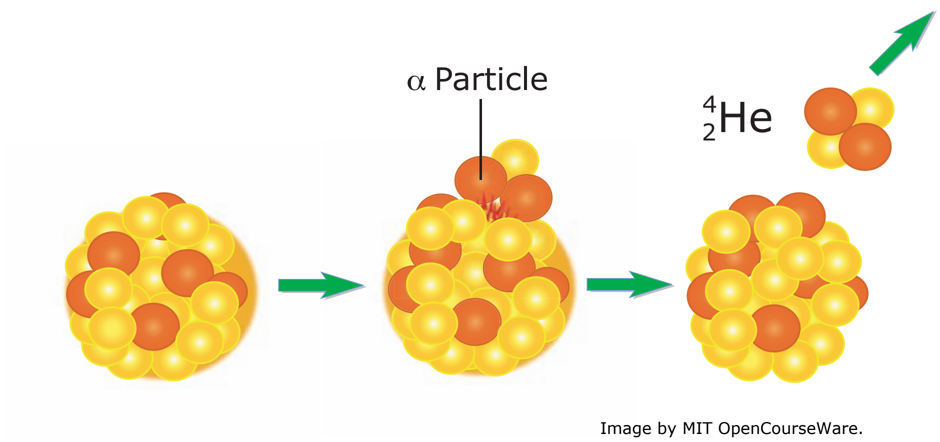

Fig. 5: Alpha decay schematics

A. Energetics

In analyzing a radioactive decay (or any nuclear reaction) an important quantity is Q, the net energy released in the decay: $Q = (m_X - m_{X'} - m_\alpha)c^2$. This is also equal to the total kinetic energy of the fragments, here $Q = T_{X'} + T_\alpha$ (here assuming that the parent nuclide is at rest).
When $Q > 0$ energy is released in the nuclear reaction, while for $Q < 0$ we need to provide energy to make the reaction happen. As in chemistry, we expect the first reaction to be a spontaneous reaction, while the second one does not happen in nature without intervention. (The first reaction is exo-energetic the second endo-energetic).
Notice that it's no coincidence that it's called Q. In practice given some reagents and products, Q give the *quality* of the reaction, i.e. how energetically favorable, hence probable, it is. For example in the alpha-decay $\log(t_{1/2}) \propto \frac{1}{\sqrt{Q_\alpha}}$, which is the Geiger-Nuttall rule (1928).
The alpha particle carries away most of the kinetic energy (since it is much lighter) and by measuring this kinetic energy experimentally it is possible to know the masses of unstable nuclides.
We can calculate Q using the SEMF. Then:

$$Q_\alpha = B({}^{A-4}_{Z-2}X'_{N-2}) + B({}^4He) - B({}^A_Z X_N) = B(A-4, Z-2) - B(A,Z) + B({}^4He)$$

We can approximate the finite difference with the relevant gradient:

$$Q_\alpha = [B(A-4,Z-2) - B(A,Z-2)] + [B(A,Z-2) - B(A,Z)] + B({}^4He) \approx = -4\frac{\partial B}{\partial A} - 2\frac{\partial B}{\partial Z} + B({}^4He)$$

$$= 28.3 - 4a_v + \frac{8}{3}a_s A^{-1/3} + 4a_c\left(1 - \frac{Z}{3A}\right)\left(\frac{Z}{A^{1/3}}\right) - 4a_{sym}\left(1 - \frac{2Z}{A} + 3a_p A^{-7/4}\right)^2$$

Since we are looking at heavy nuclei, we know that $Z \approx 0.41A$ (instead of $Z \approx A/2$) and we obtain

$$Q_\alpha \approx -36.68 + 44.9A^{-1/3} + 1.02A^{2/3},$$

where the second term comes from the surface contribution and the last term is the Coulomb term (we neglect the pairing term, since a priori we do not know if a_p is zero or not).

Then, the Coulomb term, although small, makes Q increase at large A. We find that $Q \geq 0$ for $A \gtrsim 150$, and it is $Q \approx 6$MeV for $A = 200$. Although $Q > 0$, we find experimentally that α decay only arise for $A \geq 200$.
Further, take for example Francium-200 ($^{200}_{87}\text{Fr}_{113}$). If we calculate Q_α from the experimentally found mass differences we obtain $Q_\alpha \approx 7.6$MeV (the product is ^{196}At). We can do the same calculation for the hypothetical decay into a ^{12}C and remaining fragment ($^{188}_{81}\text{Tl}_{\ 107}$):

$$Q_{^{12}C} = c^2[m(^A_Z X_N) - m(^{A-12}_{Z-6}X'_{N-6}) - m(^{12}C)] \approx 28MeV$$

Thus this second reaction seems to be more energetic, hence more favorable than the alpha-decay, yet it does not occur (some decays involving C-12 have been observed, but their branching ratios are much smaller).
Thus, looking only at the energetic of the decay does not explain some questions that surround the alpha decay:

- Why there's no ^{12}C-decay? (or to some of this tightly bound nuclides, e.g O-16 etc.)
- Why there's no spontaneous fission into equal daughters?
- Why there's alpha decay only for $A \geq 200$?
- What is the explanation of Geiger-Nuttall rule? $\log t_{1/2} \propto \frac{1}{\sqrt{Q_\alpha}}$

1.3.2 Beta decay

The beta decay is a radioactive decay in which a proton in a nucleus is converted into a neutron (or vice-versa). Thus A is constant, but Z and N change by 1. In the process the nucleus emits a beta particle (either an electron or a positron) and quasi-massless particle, the **neutrino**.

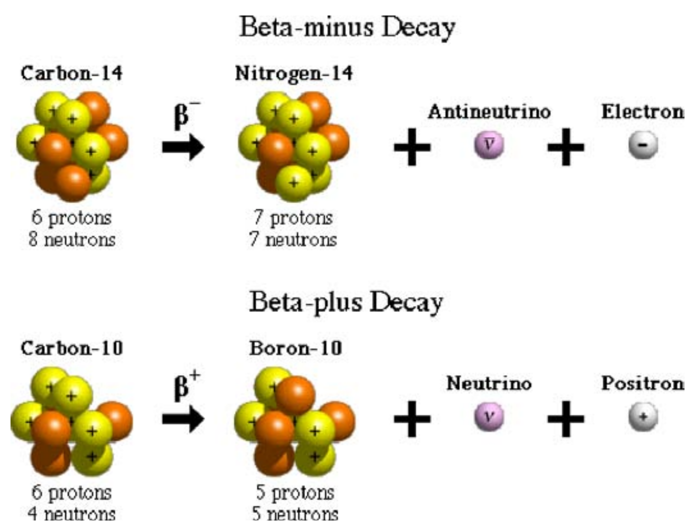

Fig. 6: Beta decay schematics

There are 3 types of beta decay:

$$^A_Z X_N \ \rightarrow \ ^A_{Z+1}X'_{N-1} + e^- + \bar{\nu}$$

This is the β^- decay (or negative beta decay). The underlying reaction is:

$$n \ \rightarrow \ p + e^- + \bar{\nu}$$

that corresponds to the conversion of a proton into a neutron with the emission of an electron and an anti-neutrino.
There are two other types of reactions, the β^+ reaction,

$$^A_Z X_N \ \rightarrow \ ^A_{Z-1}X'_{N+1} + e^+ + \nu \quad \Longleftrightarrow \quad p \ \rightarrow \ n + e^+ + \nu$$

which sees the emission of a positron (the electron anti-particle) and a neutrino; and the electron capture:

$$^A_Z X_N + e^- \ \rightarrow \ ^A_{Z-1}X'_{N+1} + \nu \quad \Longleftrightarrow \quad p + e^- \ \rightarrow \ n + \nu$$

a process that competes with, or substitutes, the positron emission.

Recall the mass of nuclide as given by the semi-empirical mass formula. If we keep A fixed, the SEMF gives the binding energy as a function of Z. The only term that depends explicitly on Z is the Coulomb term. By inspection we see that $B \propto Z^2$. Then from the SEMF we have that the masses of possible nuclides with the same mass number lie on a parabola. Nuclides lower in the parabola have smaller M and are thus more stable. In order to reach that minimum, unstable nuclides undergo a decay process to transform excess protons in neutrons (and vice-versa).

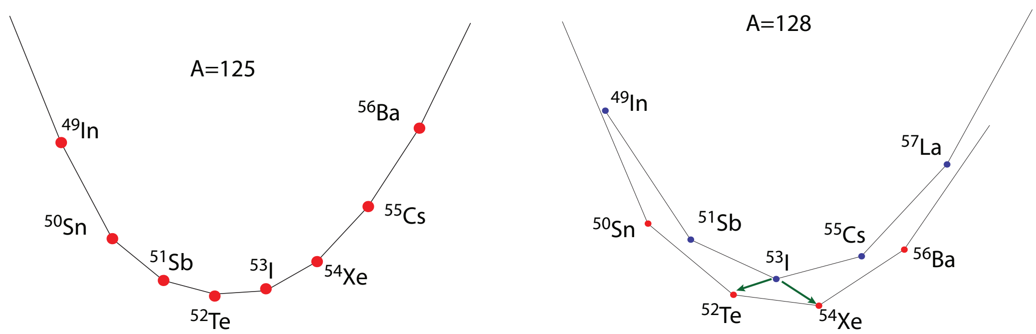

Fig. 7: Nuclear Mass Chain for A=125, (left) and A=128 (right)

The beta decay is the radioactive decay process that can convert protons into neutrons (and vice-versa). We will study more in depth this mechanism, but here we want simply to point out how this process can be energetically favorable, and thus we can predict which transitions are likely to occur, based only on the SEMF.
For example, for $A = 125$ if $Z < 52$ we have a favorable $n \to p$ conversion (beta decay) while for $Z > 52$ we have $p \to n$ (or positron beta decay), so that the stable nuclide is $Z = 52$ (tellurium).

A. Conservation laws

As the neutrino is hard to detect, initially the beta decay seemed to violate energy conservation. Introducing an extra particle in the process allows one to respect conservation of energy.
The Q value of a beta decay is given by the usual formula:

$$Q_{\beta^-} = [m_N(^AX) - m_N(^{A}_{Z+1}X') - m_e]c^2.$$

Using the atomic masses and neglecting the electron's binding energies as usual we have

$$Q_{\beta^-} = \{[m_A(^AX) - Zm_e] - [m_A(^{A}_{Z+1}X') - (Z+1)m_e] - m_e\}c^2 = [m_A(^AX) - m_A(^{A}_{Z+1}X')]c^2.$$

The kinetic energy (equal to the Q) is shared by the neutrino and the electron (we neglect any recoil of the massive nucleus). Then, the emerging electron (remember, the only particle that we can really observe) does not have a fixed energy, as it was for example for the gamma photon. But it will exhibit a spectrum of energy (or the number of electron at a given energy) as well as a distribution of momenta. We will see how we can reproduce these plots by analyzing the QM theory of beta decay.
Examples

$$^{64}_{29}\text{Cu} \begin{array}{l} \nearrow \; ^{64}_{30}\text{Zn} + e^- + \bar{\nu}, \quad Q_\beta = 0.57MeV \\ \searrow \; ^{64}_{28}\text{Ni} + e^+ + \nu, \quad Q_\beta = 0.66MeV \end{array}$$

The neutrino and beta particle ($\beta^\pm$) share the energy. Since the neutrinos are very difficult to detect (as we will see they are almost massless and interact very *weakly* with matter), the electrons/positrons are the particles detected in beta-decay and they present a characteristic energy spectrum (see Fig. 8).

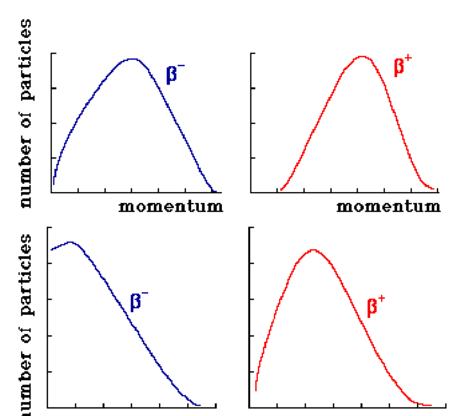

Fig. 8: Beta decay spectra: Distribution of momentum (top plots) and kinetic energy (bottom) for β^- (left) and β^+ (right) decay.

The difference between the spectrum of the $\beta^\pm$ particles is due to the Coulomb repulsion or attraction from the nucleus.

Notice that the neutrinos also carry away angular momentum. They are spin-1/2 particles, with no charge (hence the name) and very small mass. For many years it was actually believed to have zero mass. However it has been confirmed that it does have a mass in 1998.
Other conserved quantities are:
- **Momentum**: The momentum is also shared between the electron and the neutrino. Thus the observed electron momentum ranges from zero to a maximum possible momentum transfer.
- **Angular momentum** (both the electron and the neutrino have spin 1/2)
- **Parity?** It turns out that parity is not conserved in this decay. This hints to the fact that the interaction responsible violates parity conservation (so it cannot be the same interactions we already studies, e.m. and strong interactions)
- **Charge** (thus the creation of a proton is for example always accompanied by the creation of an electron)
- **Lepton number**: we do not conserve the total number of particles (we create beta and neutrinos). However the number of massive, heavy particles (or baryons, composed of 3 quarks) is conserved. Also the lepton number is conserved. Leptons are fundamental particles (including the electron, muon and tau, as well as the three types of neutrinos associated with these 3). The lepton number is +1 for these particles and -1 for their antiparticles. Then an electron is always accompanied by the creation of an antineutrino, e.g., to conserve the lepton number (initially zero).
Although the energy involved in the decay can predict whether a beta decay will occur ($Q > 0$), and which type of beta decay does occur, the decay rate can be quite different even for similar Q-values. Consider for example ^{22}Na and ^{36}Cl. They both decay by β decay:

$$^{22}_{11}\text{Na}_{11} \rightarrow ^{22}_{10}\text{Ne}_{12} + \beta^+ + \nu, \qquad Q = 0.22\text{MeV}, \quad T_{\frac{1}{2}} = 2.6\text{years}$$

$$^{36}_{17}\text{Cl}_{19} \rightarrow ^{36}_{18}\text{Ar}_{18} + \beta^- + \bar{\nu} \qquad Q = 0.25\text{MeV}, \quad T_{\frac{1}{2}} = 3 \times 10^5\text{years}$$

Even if they have very close Q-values, there is a five order magnitude in the lifetime. Thus we need to look closer to the nuclear structure in order to understand these differences.

1.3.3 Gamma decay

In the gamma decay the nuclide is unchanged, but it goes from an excited to a lower energy state. These states are called isomeric states. Usually the reaction is written as:

$$^A_Z X^*_N \longrightarrow ^A_Z X_N + \gamma$$

where the star indicate an excited state. We will study that the gamma energy depends on the energy difference between these two states, but which decays can happen depend, once again, on the details of the nuclear structure and on quantum-mechanical selection rules associated with the nuclear angular momentum.

1.3.4 Spontaneous fission

Some nuclei can spontaneously undergo a fission, even outside the particular conditions found in a nuclear reactor. In the process a heavy nuclide splits into two lighter nuclei, of roughly the same mass.

1.3.5 Branching Ratios

Some nuclei only decay via a single process, but sometimes they can undergo many different radioactive processes, that compete one with the other. The relative intensities of the competing decays are called branching ratios.
Branching ratios are expressed as percentage or sometimes as partial half-lives. For example, if a nucleus can decay by beta decay (and other modes) with a branching ration b_β, the partial half-life for the beta decay is $\lambda_\beta = b_\beta \lambda$.