DEPARTMENT OF PHYSICS AND ASTROPHYSICS
UNIVERSITY OF DELHI

B.Sc. (H) PHYSICS (LOCF)
SEMESTER – II

ELECTRICITY AND MAGNETISM

<u>Model Problem Set</u>

Questions based on,

- **Electric field and electrostatic potential**
- **Electrostatic energy of system of charges**
- **Capacitance of a system of charged conductors**
- **Method of images**

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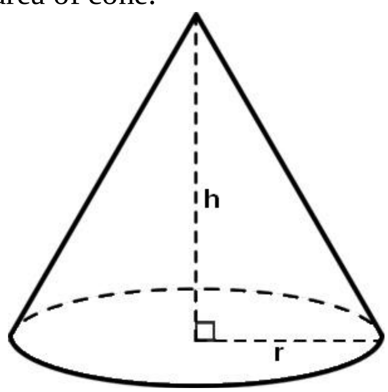
[1] A charge 'q' is placed at the centre of the base of a cone as shown in the diagram.Find the electric flux through the curved surface area of cone.



[2] A point charge 'q' is located at the centre of a cube having edge of length 'd'. What is the value of flux over one face of the cube? If thecharge is placed at one corner of the cube, then what will be the value of electric flux through each face of the cube?

[3] A spherical charge distribution has a volume charge density $\rho(r)$ given by,

$$\rho(r) = \begin{cases} \rho_0\left(1 - \frac{5r^2}{R^2}\right) & r \le R \\ 0 & r > R \end{cases}$$

Calculate the electric field at points,$r < R$and $r > R$.

[4] Calculate the electric field intensity due to a spherical charge distribution which is given by,

$$\rho(r) = \begin{cases} \rho_0\left(1 - \frac{r}{R}\right) & r < R \\ 0 & r > R \end{cases}$$

Find the value of 'r'at which the electric field is maximum.

[5] Find the electric field at the centre of,

a) A uniformly charged semicircular arc.

b) A hemisphere which is charged uniformly with a surface density σ.

[6] A finite sheet lies on the $z = 0$ plane, such that $0 \le x \le 1\text{m}$ and $0 \le y \le 1\text{m}$. It has a charge density given by,

$$\sigma = axy(x^2 + y^2 + 25\text{m}^2)^{3/2}\text{nC/m}^2$$

Here, a is a constant with appropriate units. Find the total charge on the sheet and the electric field at (0,0,5m).

[7] If the electric field is given by $\vec{E} = a[8\,\hat{\imath} + 4\,\hat{\jmath} + 3\,\hat{k}]\text{N/C}$, then calculate the electric flux through a surface of area 100 m^2 lying in the $x - y$ plane. Here, a is a constant with appropriate units.

[8] Find the potential on the axis of a uniformly charged cylinder,a distance 'z' from the centre. It is given that the length of the cylinder is L, its radius is r and the charge density is ρ.Use your result to find the electric field at this point.(Assume that $z > L/2$).

[9] An inverted hemispherical bowl of radius R carries a uniform surface charge density.Find the potential difference between North Pole and centre.

[10] A solid conducting sphere is concentric with a thin conducting shell. The inner sphere carries a charge Q_1, and the spherical shell carries a charge Q_2, such that $Q_2 = -3Q_1$.

- **a)** How is the charge distributed on the sphere?
- **b)** Calculate the charge densities distributed on the spherical shell (inner and outer surface of the shell)?
- **c)** What is the electric field at $r < R_1$, between R_1 and R_2, at $r > R_2$?
- **d)** What happens when you connect the two spheres with a wire?

[11] An isolated conductor of arbitrary shape has a net charge of $10\mu C$. There is a cavity inside the conductor within which there is a point charge of 3 μC.

- **a)** What is the charge on the cavity wall?
- **b)** What is the charge on the outer surface of the conductor?

[12] Consider a charged sphere of radius R containing charge q, completely enclosed by a spherical cavity of inner radius 'a' and outer radius 'b'. Calculate the charge density on all surfaces and potential everywhere.

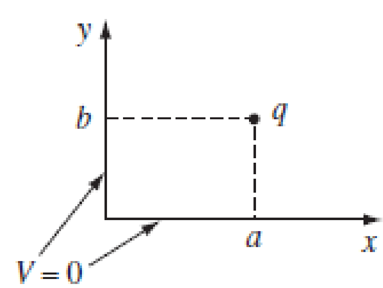
[13] A copper sphere of radius 4 cm carries a uniformly distributed total charge of 5 μC on its surface in face space.

- **a)** Use Gauss's law to find electric fluxdensity vector D external to the sphere.
- **b)** Calculate the total energy stored in the electrostatic field.
- **c)** Calculate the capacitance of the isolated sphere.

[14] A point charge 'q' is placed inside a hollow grounded, conducting sphere of inner radius 'a'. Using the method of images,

- **a)** Find the potential inside the sphere
- **b)** Find the induced surface-charge density.

[15] Two semi-infinite grounded conducting planes



meet at right angles. In the region between them, there is a point charge 'q' as shown in the figure. Calculate the

a) potential in the region of charge 'q'

b) induced charged densities and total induced charged on the two planes

c) total electrostatic energy of the system

d) suppose the planes met at some angle other than 90^{o}, would you still be able to solve the problem by the method of images? If not, for what particular angles does the method work?

[16] Two large parallel grounded conducting plates are separated by a small distance $4d$. If two charges Q and $-Q$ are placed at distances d and $3d$ from one plate in the middle. Calculate

a) The total charged induced on the plates

b) Force and magnitude on each charge

c) The total charged induced on each plate if $-Q$ is removed.

[17] Three identical point charges of $4\,pC$ each are located at the corners of equilateral triangle having each side of 0.5 mm. How much work must be done to move one charge to a point equidistant from the other two and on the line joining them?

[18] Two spherical cavities, of radii 'a' and 'b', are hollowed out from the interior of a neutral conducting sphere of radius R. At the center of each cavity point charge q_a and q_b are placed respectively.

a) Find the surface charges densities σ_a, σ_b and σ_R.

b) What is the field outside the conductor?

c) What is the field outside the conductor and within each cavity

d) What is the force on q_a and q_b?

Questions based on,

- **Dielectric Properties of Matter**
- **Capacitor filled with dielectric**
- **Gauss law in dielectrics**

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[1] In a parallel plate air capacitor having plate separation 0.04 mm, an electric field of 4×10^4 V/m is established between the plates. The battery is then removed and a metal plate of thickness 0.03 mm is inserted between the plates of the capacitor. Determine the potential difference across the capacitor,

a) Before the introduction of metal plates

b) After the introduction of metal plates

c) If dielectric slab with dielectric constant 2.5 and same thickness is inserted instead of the metal plates

[2] The space between the plates of a parallel plate capacitor (kept at 'd' distance apart and each having area A) is filled with a dielectric, whose relative permittivity varies according with distance (x) according to the relation,

$$\varepsilon_r = \frac{1}{1 + \frac{x^2}{d^2}}$$

Find the capacitance of the capacitor.

[3] An isolated conducting sphere of radius 'a' is enclosed within an earthed concentric sphere of radius 'b'. Show that capacitance of this system will be 'n' times that of an isolated sphere, if

$$\frac{b}{a} = \frac{n}{n-1}$$

[4] A 4 μF capacitor has a charge of 40 μC and a 3 μF capacitor has a charge of 10 μC. The negative plate of each one is connected to the positive plate of the other. Determine,

- **a)** The charge on each capacitor
- **b)** The potential difference across each capacitor

[5] A plane slab of isotropic material of relative permittivity $\varepsilon_r = 5$ is placed normal to a uniform electric field with an electric displacement vector of magnitude 5 cm^2. If the slab occupies a volume of $0.5m^2$ and is uniformly polarized, find (a) the magnitude of **P** in slab and (b) the magnitude of the total dipole moment of the slab.

[6] A 4 μF capacitor has a charge of 40 μC and a 3 μF capacitor has a charge of 10 μC. The capacitors are connected such that plates of like charge are connected together. Determine,

- **a)** The initial potential difference across each capacitor
- **b)** The final potential difference across each capacitor
- **c)** The initial energy of each capacitor
- **d)** The final energy of the combination
- **e)** The energy lost in connecting them together.

Questions based on,

- **Magnetic Field**
- **Biot-Savart Law and its applications**
- **Ampere's Circuital Law and its application**
- **Vector Potential**
- **Magnetic Properties of Matter**

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[1] A current I flows down a wire of radius R.

- **a)** If it is uniformly distributed over the surface, what will be the current density?
- **b)** If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, then what will be the current density?

[2] Consider three straight, infinitely long, equally spaced wires (zero radius), each carrying a current I in the same direction.

- **a)** Calculate the location of points of zero magnetic field.
- **b)** Sketch the magnetic field pattern.

[3] Two coaxial solenoids are each carrying a current I, but in opposite directions. The innersolenoid of radius 'a' has N_1turns per unit length and the outer solenoid of radius 'b' has N_2 turns per unit length. Find **B**in each of the three regions,

a) Inside the inner solenoid
b) Between the solenoids
c) Outside both the solenoids

[4] An infinite solenoid having N turns per unit length and current 'I'is filled with a linearmagnetic material.

a) Find the magnetic field inside the solenoid.
b) Find the bound surface current density.
c) Explain what will be the difference in the field found above, if the magnetic material is paramagnetic or diamagnetic.

[5] Consider a large parallel plate capacitor with uniform surface charge density σ onthe upper plate and $-\sigma$ on the lower plate. Both the plates are together moving towardsright with a constant speed v. Find the magnetic field between, above and below thetwo plates of the capacitor.

[6] At the equator, the strength of the earth's magnetic field is ~ 0.03 mWb/m^2. If a piece of copper wire 1 m long and cross-sectional area$3 \times 10^{-6}\text{m}^2$ is to be kept afloatin a horizontal position perpendicular to the direction of the magnetic field, how muchcurrent needs to be passed through the wire. Given that density of copper is $9 \times 10^3\text{kg/m}^3$

[7] In the Rowland ring 2.0 amp current is passing through the winding of number of turns per unit length is 10 turns/cm and the magnetic induction measured is 1.0 wb/m^2. Calculate (a) **H** (b) **M** and (c) the magnetizing current, both when the core is placed and when it is removed.

[8] An iron rod 20 cm long, 1 cm in diameter and of a permeability 1000 is placed inside a solenoid 1 m long wound uniformly with 600 turns. If the current of 0.5 ampere is passed through the solenoid, find the magnetic moment of the rod.

[9] An electron moving speed $v = 3 \times 10^6 \text{m/s}$ enters the gap between the plates of a parallel plate capacitor, midway between the plates with its velocity parallel to the plates.The plates are separated by a distance of 1cm, the potential difference between the plates is 100 V and the length of the plates is 30 cm. A fluorescent screen is placed at a distance$D$ from the further end of the plates. Determine the distance from the midpoint of thescreen where the electron will hit. Assume that the field is non zero only between theplates and neglect edge effects.

[10] A very thin disc of radius R, carrying a uniform surface charge density σ lies parallelto the $x - y$ plane. It rotates about the z-axis(passing through its center) with an angularspeed ω. Find the direction and magnitude of the magnetic field at the center of thedisk.

[11] A circular current carrying coil of radius R is in the $x - y$ plane with its centre at $z = 0$. The magnetic field at point P on the z-axis at a distance 'z' above the centre iswell known. Using the expression and the fact that$\vec{\nabla} \cdot \vec{B} = 0$, obtain the value of $B_x$and B_y at a point Q away from the axis the same value of z as P but removed by smallvalues of $x$and y. Retain terms only up to first order in $x$and y.

[12] In a B-H hysteresis loop $B_{max.}$ = 1.375 wb/m^2, the area of the loop is 0.513 sq. inch and the coordinates are such that 1'' along x-axis = 10^{-3} amp. turns/cm and 1'' along y-axis = 1 wb/m^2. Find the loss of energy in watts due to hysteresis if 10^{-3} of iron were subjected to a flux density of 13.75 wb/m^2 at 25 cycles/sec.

Questions based on,

- **Electromagnetic Induction**
- **Self Inductance and Mutual Inductance**
- **Charge Conservation and Displacement current**

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[1] A 50 cm long wire is moved in the y-direction at a speed of 25 m/sec, so that the wire is always parallel to the x-axis. The components of the magnetic field are,

$$B_x = 0.2\ \mathrm{Wb/m^2}$$
$$B_y = -0.4\ \mathrm{Wb/m^2}$$
$$B_z = 0.3\ \mathrm{Wb/m^2}$$

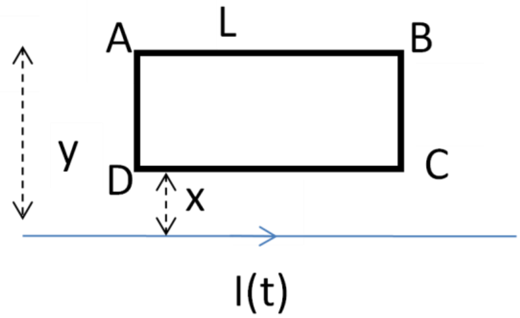
Find the e.m.f induced in the wire.

[2] A long cylindrical solenoid with 50 loops per 1 cm has a radius of 1.5 cm. Assume that the magnetic field inside the solenoid is homogeneous and parallel to the axis of the solenoid.

a) What is the inductance of the solenoid per 1 meter of its length?

b) What is the induced electromotive force per 1 meter of the length of solenoid if the current is changing at a rate of 10 A/s ?

[3] Consider an infinite long wire carrying current I(t), with dI/dt = λ = constant Find the current produced in the rectangular loop of wire ABCD, if its resistance is R.



[4] A small circular loop, of radius r, is kept parallel to a big circular loop, of radius, R ($R >> r$), such that its center is along the axis of the big loop. Distance between the centers of the two loops is d. Determine the coefficient of mutual inductance, due to a current, I, flowing through the loops.

[5] The conductivity σ of a material is 5 (Ωm)$^{-1}$ and ε_r= 1. The electric field varies according to equation **E** = 200 sin 10^9t Vm^{-1}. Calculate the conduction and displacement current densities. At what frequency the two are equal?

[6] Assuming sea water has $\mu = \mu_0, \varepsilon = 81\varepsilon_0, \sigma = 20\ \text{S/m}$, determine the frequency at which magnitude of conduction current density is 5 times that of the displacement current density.

Questions based on,

- **Electrical Circuits**
- **Series and parallel LCR Circuit**
- **Network theorems**
- **Mesh & Node Analysis**

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[1] A series LCR circuit with $L = 25\text{nH}$, $C = 70\mu\text{F}$ has a lagging phase angle of 20° at $\omega = 2\text{kHz}$. At what frequency will the phase angle be leading 30°?

[2] A series LCR circuit has $R = 4\Omega, L = 0.5\ H, V = 100\cos 50\pi t$. Find the value of C for resonance. Also determine the voltage across C.

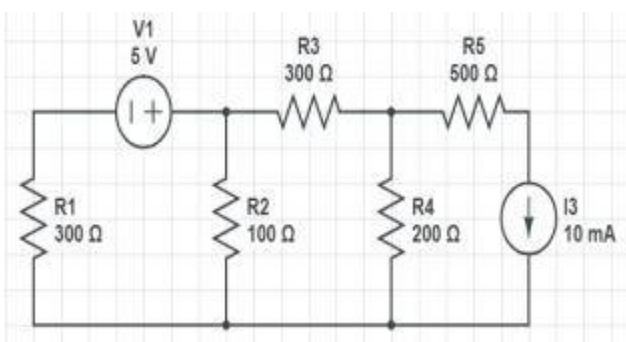
[3] An emf of 230 V rms at 50 Hz is applied to a circuit containing a capacitance of 5 μF in series with a resistance of 1000 Ω. Find the rate at which energy is dissipated in the circuit.

[4] A 10 Henry coil of resistance 50 Ω is connected in series with 0.5 μF capacitor. The combination is subjected to an alternating voltage of 240 V 50 Hz. Calculate the power dissipated in the coil and the power factor.

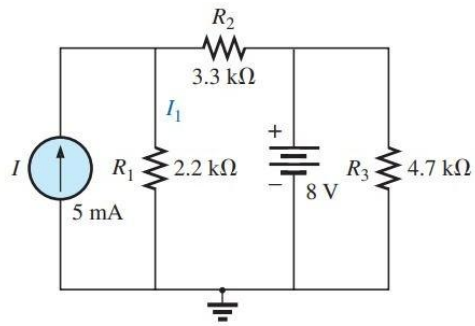
[5] A circuit with $R = 50\ \Omega$ in series with a parallel combination of $L = 2$ mH and $C = 30\mu$F is driven by 230 V supply with angular frequency $\omega = 300$ rad/sec. (a) Obtain the impedance, (b) Rms value current (c) Explain the behavior of circuit at frequency $\omega_o = 1/\sqrt{(LC)}$.

[6] A coil of inductance 2 mH and resistance R is connected in parallel with a capacitor of magnitude 0.001 μF. (a) Find the value of R at frequency $f = 113 \times 10^3$ cycle/sec for minimum current in the circuit. (b) The peak value of make-up current if peak supply emf is 2V.

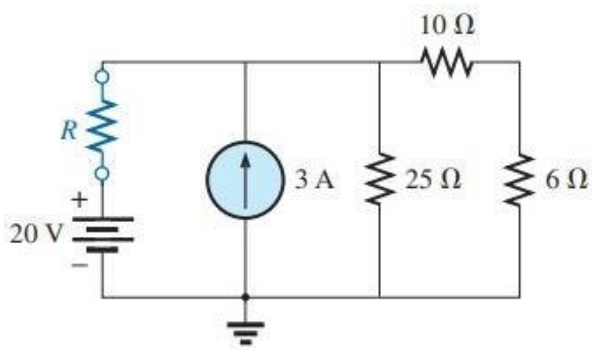
[7] Use Mesh and Nodal Analysis to find the voltage across 1Ω resistance



[8] Use the superposition theorem to find current through R1 for the given network

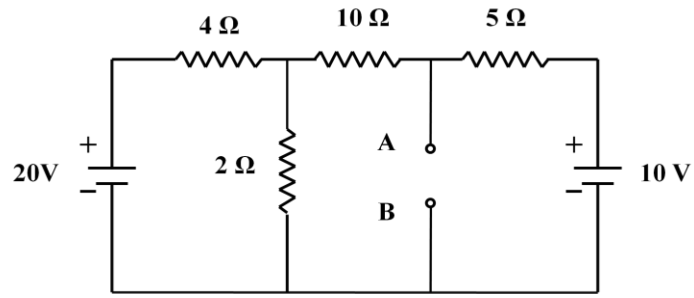


[9] Obtain Thevenin equivalent circuit for the network external to the resistor R for the givennetwork.

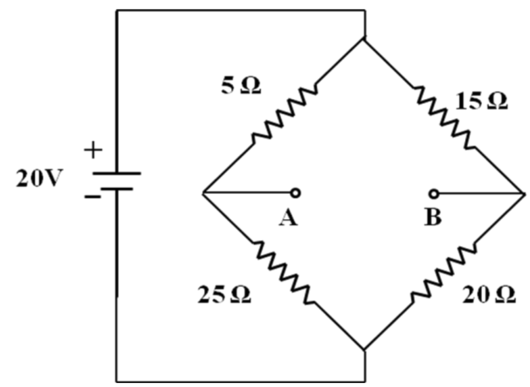


[10] Obtain the Thevenin's equivalent circuit for the following circuits,

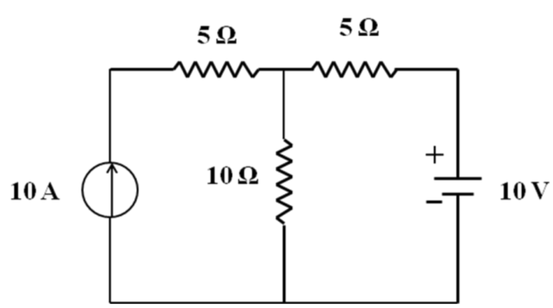
(i)



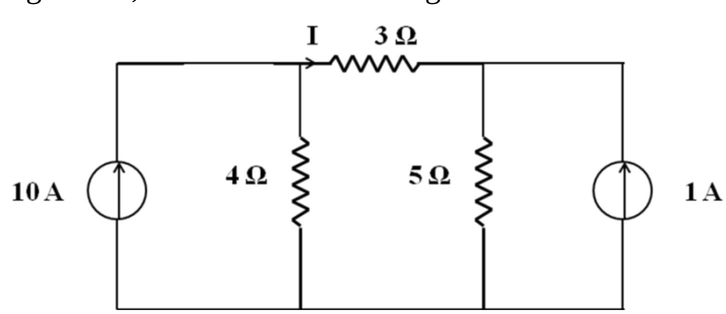
(ii)



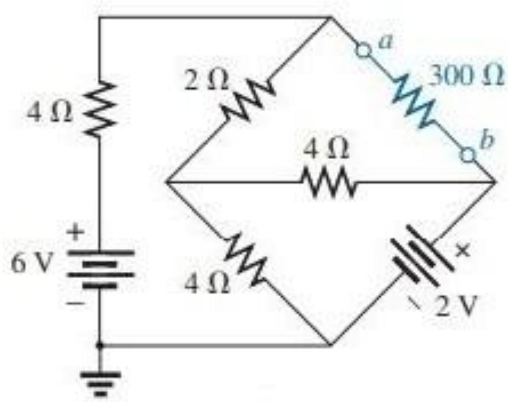
[11] Find the current flowing through the 10 Ω resistance in the following circuit.



[12] For the following circuit, find the current I using the Norton's theorem.



[13] Obtain Norton equivalent circuit for the network external to the 300 Ω resistance for the given network.



[14] In the following circuit, the total power dissipated is 64 W. Calculate the value of the resistance R.