Piecewise interpolation → (i) linear → x: $x_0 \quad x_1 \quad x_2 \quad --- \quad x_n$

linear

$f(x): f(x_0), f(x_1), f(x_2) \;---\; f(x_n)$

Let the given interval $[a, b]$ where $a = x_0 < x_1 < x_2 < --- < x_n = b$

In the interval $[x_{i-1}, x_i]$

the piecewise linear interpolating polynomial =

$$\boxed{P_{i,1}(x) = \frac{x - x_i}{x_{i-1} - x_i} f(x_{i-1}) + \frac{x - x_{i-1}}{x_i - x_{i-1}} f(x_i)} \quad i = 1, 2, --- n$$

e.g. obtain the piecewise linear interpolating poly. for the fn $f(x)$ defined by the data.

	x_0	x_1	x_2	x_3
x :	1	2	4	8
$f(x)$:	3	7	21	73

Hence the estimate the values of $f(3)$ and $f(7)$

Soln in the interval $[1, 2]$

$$P_{1,1}(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$= \frac{x-2}{(-1)}(3) + \frac{x-1}{2-1}(7) = 4x - 1$$

In the interval $[2, 4]$

$$P_{2,1}(x) = \frac{x - x_2}{x_1 - x_2} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2)$$

$$= \frac{x-4}{-2}(7) + \frac{(x-2)}{2}(21) = 7x - 7$$

$[4, 8]$

$$P_{3,1}(x) = \frac{x-8}{-4}(21) + \frac{(x-4)}{4}(73) = 13x - 31$$

Piecewise linear interpolating Polynomials are given as

$$P_1(x) = \begin{cases} 4x - 1, & 1 \le x \le 2 \\ 7x - 7, & 2 \le x \le 4 \\ 13x - 31, & 4 \le x \le 8 \end{cases}$$

$f(3) = 7 \times 3 - 7 = 14$

$f(7) = 91 - 31 = 60$

e.g. x : 0, 1, 2, 3
$f(x)$: 1, 2, 5, 10

find piece wise linear interpolating poly..

and value $f(0.5)$, $f(1.5)$, $f(2.5)$

e.g. x : 0.5, 1.5, 2.5
$f(x)$: 0.125, 3.375, 15.625

find $f(1)$ and $f(2)$.

Piecewise quadratic interpolation :→ let the no. of distinct points be $2n+1$ with $a = x_0 < x_1 < x_2 < x_3 < --- < x_{2n} = b$

Consider the group of three consecutive points as

$[x_0\ x_2], [x_2\ x_4], --- [x_{i-1}, x_{i+1}] -- [x_{2n-2}, x_{2n}]$

the quadratic interpolation polynomial. for $[x_{i-1}, x_{i+1}]$

$$P_{i,2}(x) = \frac{(x-x_i)(x-x_{i+1})}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})} f(x_{i-1}) + \frac{(x-x_{i-1})(x-x_{i+1})}{(x_i-x_{i-1})(x_i-x_{i+1})} f(x_i)$$
$$+ \frac{(x-x_{i-1})(x-x_i)}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)} f(x_{i+1})$$

e.g. obtain the piecewise quadratic interpolating poly. for the f^n $f(x)$ defined the data.

x : -3, -2, -1, 1, 3, 6, 7
$f(x)$: 369, 222, 171, 165, 207, 990, 1779

and find $f(-2.5)$ and $f(6.5)$

Sol^n :→ on $[-3, -1]$ $\qquad x_{i-1} = -3$, $x_i = -2$, $x_{i+1} = -1$

$$P_{1,2}(x) = \frac{(x+2)(x+1)}{(-3+2)(-3+1)} 369 + \frac{(x+3)(x+1)}{(-2+3)(-2+1)}(222) + \frac{(x+3)(x+2)}{(-1+3)(-1+2)}(171)$$

$$= \frac{369}{2}(x^2+3x+2) - 222(x^2+4x+3) + \frac{171}{2}(x^2+5x+6)$$

$$= 48x^2 + 93x + 216$$

on $[-1, 3]$ $P_{2,2}(x) = 6x^2 - 3x + 162$

on $[3, 7]$ $P_{3,2}(x) = 132x^2 - 927x + 1800$

$f[-2.5] = 48(-2.5)^2 + 93(-2.5) + 216 = 283.5$

$f[6.5] = 132(6.5)^2 - 927(6.5) + 1800 = 1351.5$

e.g. $x: -2, 0, 1, 3, 4$

$f(x): -23, 1, 4, 82, 193$

find $f(0.5)$ and $f(2.0)$

e.g. $x: -2, -1, 1, 2, 4$

$f(x): -29, -8, -2, -5, 7$

find $f(0.5)$ and $f(3.0)$

e.g. $x: -5, -4, -2, 0, 1, 3, 4$

$f(x): 275, -94, -334, -350, -349, -269, -94$

find $f(-3.0)$ and $f(2)$

e.g. $x: -4, -3, -1, 1, 2, 4, 5$

$f(x): -108, -255, -303, -303, -300, -108, 225$

find $f(0)$ and $f(3)$

Piecewise cubic interpolation :- let the no. of distinct points be $3n+1$ with $a_0 = x_0 < x_1 < x_2 < \text{———} < x_{3n} = b$

we consider groups of four points as $[x_0 \cdots x_3]$ $[x_3 - - x_6]$ - - —— $[x_{3n-3} - - x_{3n}]$.

for $x \in [x_i, x_{i+3}]$ the cubic interpolating polynomial.

$$P_{i,3}(x) = \frac{(x-x_{i+1})(x-x_{i+2})(x-x_{i+3})}{(x_i-x_{i+1})(x_i-x_{i+2})(x_i-x_{i+3})} f(x_i) + \frac{(x-x_i)(x-x_{i+2})(x-x_{i+3})}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})(x_{i+1}-x_{i+3})} f(x_{i+1})$$

$$+\frac{(x-x_i)(x-x_{i+1})(x-x_{i+3})}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})(x_{i+2}-x_{i+3})}f(x_{i+2})+\frac{(x-x_i)(x-x_{i+1})(x-x_{i+2})}{(x_{i+3}-x_i)(x_{i+3}-x_{i+1})(x_{i+3}-x_{i+2})}f(x_{i+3})$$

e.g. obtain the piecewise quadratic interpolating poly. for the f^n $f(x)$ defined by the data,

x : $-3,\ -2,\ -1,\ 1,\ 3,\ 6,\ 7$

$f(x)$: $369,\ 222,\ 171,\ 165,\ 207,\ 990,\ 1779$

find $f(-2.5)$ & $f(6.5)$

$\Downarrow$ 280.5 $\qquad$ $\Downarrow$ 1339.25

$P_{1,3}(x) = -8x^3+5x+168$

$P_{2,3}(x) = 14x^3-92x^2+207x+36$

e.g. x : $-5,\ -4,\ -2,\ 0,\ 1,\ 3,\ 4$

$f(x)$: $275,\ -94,\ -334,\ -350,\ -349,\ -269,\ -94$

find $f(-3.0)$ and $f(2.0)$

e.g. x ; $-4,\ -3,\ -1,\ 1,\ 2,\ 4,\ 5$

$f(x)$: $-108,\ -255,\ -303,\ -303,\ -300,\ -108,\ 225$

find $f(0)$ & $f(3)$