## (1) Newton's Forward Difference interpolation Formula :→

$x : \quad x_0 \quad x_1 \quad x_2 \quad x_3 \quad \text{———} \quad x_n$

$y : \quad y_0 \quad y_1 \quad y_2 \quad y_3 \quad \text{———} \quad y_n$

$x_1 = x_0 + h \,,\; x_2 = x_0 + 2h \quad \text{— —} \quad x_n = x_0 + nh$

$h$ = interval

$$y = f(x) = y_0 + P\,\Delta y_0 + \frac{P(P-1)}{\lfloor 2}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{\lfloor 3}\Delta^3 y_0 + \text{——}$$

$$\frac{P(P-1)(P-2)(P-3)}{\lfloor 4}\Delta^4 y_0 + \text{—— ——}$$

where $P = \dfrac{x - x_0}{h}$ i.e $x = x_0 + Ph$.

### Formulation of forward difference table :→

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| $x_0$ | $y_0$ | $\Delta y_0 = y_1 - y_0$ | $\Delta^2 y_0 = \Delta y_1 - \Delta y_0$ | $\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$ | $\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$ |
| $x_1$ | $y_1$ | $\Delta y_1 = y_2 - y_1$ | $\Delta^2 y_1 = \Delta y_2 - \Delta y_1$ | $\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$ | |
| $x_2$ | $y_2$ | $\Delta y_2 = y_3 - y_2$ | $\Delta^2 y_2 = \Delta y_3 - \Delta y_2$ | | |
| $x_3$ | $y_3$ | $\Delta y_3 = y_4 - y_3$ | | | |
| $x_4$ | $y_4$ | | | | |

e.g. Find the cubic poly. which takes following values.

$x : \quad 0, \quad 1, \quad 2, \quad 3$

$f(x) : \quad 1, \quad 2, \quad 1, \quad 10$

Also find ~~[illegible]~~ and ~~[illegible]~~ $f(4)$ and $f'(4)$, $\quad f(2.5)$ and $f'(2.5)$

$= 3.5$

Sol$^n$ the Forward difference table. for given data points

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| $x_0 = 0$ | $y_0 = 1$ | $\Delta y_0 = 2-1 = 1$ | $\Delta^2 y_0 = -2$ | $\Delta^3 y_0 = 10+2 = 12$ |
| 1 | 2 | $1-2 = -1$ | 10 | |
| 2 | 1 | $10-1 = 9$ | | |
| 3 | 10 | | | |

Here $x_0 = 0$ , $y_0 = 1$ , $h = 1$ $\Rightarrow$ $p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$ (42)

Now by Newton's Forward difference interpolation formula

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{\lfloor 2} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3} \Delta^3 y_0$$

$$= 1 + x.1 + \frac{x(x-1)}{\lfloor 2} (-2) + \frac{x(x-1)(x-2)}{\lfloor 3} .12$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

$$f'(x) = 6x^2 - 14x + 6$$

$f(4) = 41$

$f'(4) = 46$

~~e.g.~~ from the table, estimate the no. of students who obtained marks between 40 and 45

| marks : | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 |
|---|---|---|---|---|---|
| no. of students : | 31 | 42 | 51 | 35 | 31 |

marks below x | no. of students y |

e.g.

| $x$ : | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| $y = f(x)$ : | 1 | 1.6 | 3.8 | 8.2 | 15.4 |

find $f(3)$

Sol$^n$ $\Rightarrow$

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 0 | 1 | | | | |
| | | .6 | | | |
| 5 | 1.6 | | 1.6 | | |
| | | 2.2 | | .6 | |
| 10 | 3.8 | | 2.2 | | 0 |
| | | 4.4 | | .6 | |
| 15 | 8.2 | | 2.8 | | |
| | | 7.2 | | | |
| 20 | 15.4 | | | | |

$h = 5$, $x = 3$ , $x_0 = 0$

$$p = \frac{3 - 0}{5} = .6$$

$$y = f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{\lfloor 2} \Delta^2 y_0 + \text{\_\_\_\_\_\_}$$

$= 1 +$ ______

$y = \underline{1.2016}$

e.g. $x$ : 0.1 , 0.2 , 0.3, 0.4, 0.5

$f(x)$ : 1.40 , 1.56 , 1.76 , 2.00 , 2.28

find $f(x)$ ~~and~~, $f(0.25)$ and $f(0.35)$

Sol$^n$ $f(x) = 2x^2 + x + 1.28$

$f(0.25) = 1.655$ , $f(0.35) = 1.875$

e.g. $x$ : 0 , 0.1 , 0.2 , 0.3 ; 0.4 , 0.5

$y$ : −1.5 , −1.27 , −0.98 , −0.63 , −0.22 , 0.25

find $f(x)$ and $f(0.15)$

## Newton - Backward difference formula :-

$x$ : $x_0$ $x_1$ $x_2$ ——— $x_n$

$y$ : $y_0$ $y_1$ $y_2$ ——— $y_n$

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{\lfloor 2}\nabla^2 y_n + \frac{p(p+1)(p+2)}{\lfloor 3}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{\lfloor 4}\nabla^4 y_n + \text{———}$$

where $p = \dfrac{x - x_n}{h}$

new: Backward difference table :-

| $x$ | $y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ |
|---|---|---|---|---|---|
| $x_0$ | $y_0$ | | | | |
| $x_1$ | $y_1$ | $\nabla y_1 = y_1 - y_0$ | | | |
| $x_2$ | $y_2$ | $\nabla y_2 = y_2 - y_1$ | $\nabla^2 y_2 = \nabla y_2 - \nabla y_1$ | $\nabla^3 y_3 =$ | $\nabla^4 y_4 =$ |
| $x_3$ | $y_3$ | $\nabla y_3 = y_3 - y_2$ | $\nabla^2 y_3 = \nabla y_3 - \nabla y_2$ | $\nabla^3 y_4 =$ | |
| $x_4$ | $y_4$ | $\nabla y_4 = y_4 - y_3$ | $\nabla^2 y_4 = \nabla y_4 - \nabla y_3$ | | |

e.g. $x$ : 20 , 25 , 30 , 35 , 40 , 45

$f(x)$ : 354 , 332 , 291 , 260 , 231 , 204

find $f(42)$

Solⁿ the Backward difference table.

| $x$ | $f(x)$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ | $\nabla^5 y$ |
|---|---|---|---|---|---|---|
| 20 | 354 | | | | | |
| | | -22 | | | | |
| 25 | 332 | | -19 | | | |
| | | -41 | | 29 | | |
| 30 | 291 | | 10 | | -37 | |
| | | -31 | | -8 | | 45 |
| 35 | 260 | | 2 | | 8 | |
| | | -29 | | 0 | | |
| 40 | 231 | | 2 | | | |
| | | $\nabla y_n = -27$ | | | | |
| 45 | $y_n = 204$ | | | | | |

let $x = 42$, $h = 5$, $x_n = 45$ $\quad P = \frac{x - x_n}{h} = \frac{42-45}{5} = -\frac{3}{5} = -0.6$

by N. B. D. Formula.

$$f(x) = y_n + P\nabla y_n + \frac{P(P+1)}{\lfloor 2}\nabla^2 y_n + \frac{P(P+1)(P+2)}{\lfloor 3}\nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{\lfloor 4}\nabla^4 y_n + \frac{P(P+1)(P+2)(P+3)(P+4)}{\lfloor 5}\cdot\nabla^5 y_n$$

$$\boxed{f(42) = 219}$$

e.g. find $f(7.5)$

$x$ : 1, 2, 3, 4, 5, 6, 7, 8

$y$ : 1, 8, 27, 64, 125, 216, 343, 512

Solⁿ $x_n = 8$, $y_n = 512$, $h = 1$, $x = 7.5$

Ans $f(7.5) = 421.875$

e.g. $x$ : 0, 5, 10, 15, 20

$f(x)$ : 1.0, 1.6, 3.8, 8.2, 15.4

find $f(17) = ?$

Solⁿ

| $x$ | $f(x)$ | $\nabla y_n$ | $\nabla^2 y_n$ | $\nabla^3 y_n$ | $\nabla^4 y_n$ |
|---|---|---|---|---|---|
| 0 | 1.0 | | | | |
| | | .6 | | | |
| 5 | 1.6 | | 1.6 | | |
| | | 2.2 | | 0.6 | |
| 10 | 3.8 | | 2.2 | | 0 |
| | | 4.4 | | 0.6 | |
| 15 | 8.2 | | 2.8 | | |
| | | 7.2 | | | |
| 20 | 15.4 | | | | |

$x = 17$, $x_n = 20$ $h = 5$

$f(17) = \cancel{8.2 + 1.76 + 0.616 + 0.1344}$

$= 10.7104$

e.g. $x$ : 0.1 , 0.3 , 0.5 , 0.7 , 0.9 , 1.1

$f(x)$ : −1.699 , −1.073 , −0.375 , 0.443 , 1.429 , 2.631

find $f(x)$ at $x = 0.6$

e.g. The following data represents the $f^n$ $f(x) = e^x$

$x$ : 1 , 1.5 , 2.0 , 2.5

$f(x)$ : 2.7183 , 4.4817 , 7.3891 , 12.1825

Find the ~~value find~~ value of $f(2.25)$ using (i) Newton's forward difference interpolation

(ii) Newton's backward difference interpolation.

Compare with the exact value.

e.g. $f(x) = \cos(x+1)$

$x$ : 0.0 , 0.2 , 0.4 , 0.6

$f(x)$ : 0.5403 , 0.3624 , 0.1700 , −0.0292

Estimate $f(0.5)$ using N. B. D. interpolation. Compare with the exact value.

~~0.11 − 460.3512~~