If information bits are encoded (grouped) and then converted to signals with more than two levels, transmission rates in excess of 2*B* are possible, as will be seen in subsequent sections of this chapter. In addition, since baud is the encoded rate of change, it also equals the bit rate divided by the number of bits encoded into one signaling element. Thus,

$$\text{baud} = \frac{f_b}{N} \tag{11}$$

By comparing Equation 10 with Equation 11, it can be seen that with digital modulation, the baud and the ideal minimum Nyquist bandwidth have the same value and are equal to the bit rate divided by the number of bits encoded. This statement holds true for all forms of digital modulation except frequency-shift keying.

3 AMPLITUDE-SHIFT KEYING

The simplest digital modulation technique is *amplitude-shift keying* (ASK), where a binary information signal directly modulates the amplitude of an analog carrier. ASK is similar to standard amplitude modulation except there are only two output amplitudes possible. Amplitude-shift keying is sometimes called *digital amplitude modulation* (DAM). Mathematically, amplitude-shift keying is

$$v_{(ask)}(t) = [1 + v_m(t)]\left[\frac{A}{2}\cos(\omega_c t)\right] \tag{12}$$

where
$v_{ask}(t)$ = amplitude-shift keying wave
$v_m(t)$ = digital information (modulating) signal (volts)
$A/2$ = unmodulated carrier amplitude (volts)
ω_c = analog carrier radian frequency (radians per second, $2\pi f_c t$)

In Equation 12, the modulating signal ($v_m[t]$) is a normalized binary waveform, where +1 V = logic 1 and −1 V = logic 0. Therefore, for a logic 1 input, $v_m(t) = +1$ V, Equation 12 reduces to

$$v_{(ask)}(t) = [1 + 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$
$$= A\cos(\omega_c t)$$

and for a logic 0 input, $v_m(t) = -1$ V, Equation 12 reduces to

$$v_{(ask)}(t) = [1 - 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$
$$= 0$$

Thus, the modulated wave $v_{ask}(t)$, is either $A\cos(\omega_c t)$ or 0. Hence, the carrier is either "*on*" or "*off*," which is why amplitude-shift keying is sometimes referred to as *on-off keying* (OOK).

Figure 2 shows the input and output waveforms from an ASK modulator. From the figure, it can be seen that for every change in the input binary data stream, there is one change in the ASK waveform, and the time of one bit (t_b) equals the time of one analog signaling element (t_s). It is also important to note that for the entire time the binary input is high, the output is a constant-amplitude, constant-frequency signal, and for the entire time the binary input is low, the carrier is off. The bit time is the reciprocal of the bit rate and the time of one signaling element is the reciprocal of the baud. Therefore, the rate of change of the

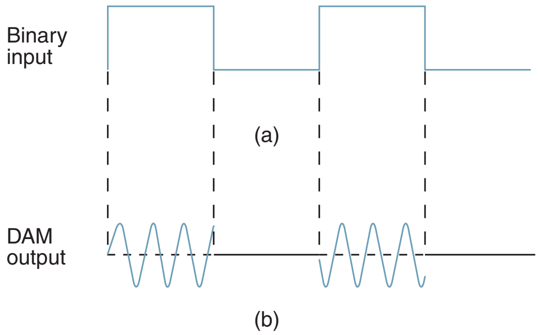


FIGURE 2 Digital amplitude modulation: (a) input binary; (b) output DAM waveform

ASK waveform (baud) is the same as the rate of change of the binary input (bps); thus, the bit rate equals the baud. With ASK, the bit rate is also equal to the minimum Nyquist bandwidth. This can be verified by substituting into Equations 10 and 11 and setting N to 1:

$$B = \frac{f_b}{1} = f_b \qquad \text{baud} = \frac{f_b}{1} = f_b$$

Example 1

Determine the baud and minimum bandwidth necessary to pass a 10 kbps binary signal using amplitude shift keying.

Solution For ASK, $N = 1$, and the baud and minimum bandwidth are determined from Equations 11 and 10, respectively:

$$B = \frac{10{,}000}{1} = 10{,}000$$

$$\text{baud} = \frac{10{,}000}{1} = 10{,}000$$

The use of amplitude-modulated analog carriers to transport digital information is a relatively low-quality, low-cost type of digital modulation and, therefore, is seldom used except for very low-speed telemetry circuits.

4 FREQUENCY-SHIFT KEYING

Frequency-shift keying (FSK) is another relatively simple, low-performance type of digital modulation. FSK is a form of constant-amplitude angle modulation similar to standard frequency modulation (FM) except the modulating signal is a binary signal that varies between two discrete voltage levels rather than a continuously changing analog waveform. Consequently, FSK is sometimes called *binary FSK* (BFSK). The general expression for FSK is

$$v_{fsk}(t) = V_c \cos\{2\pi[f_c + v_m(t)\,\Delta f]t\} \tag{13}$$

where $v_{fsk}(t)$ = binary FSK waveform
V_c = peak analog carrier amplitude (volts)
f_c = analog carrier center frequency (hertz)
Δf = peak change (shift) in the analog carrier frequency (hertz)
$v_m(t)$ = binary input (modulating) signal (volts)

From Equation 13, it can be seen that the peak shift in the carrier frequency (Δf) is proportional to the amplitude of the binary input signal ($v_m[t]$), and the direction of the shift

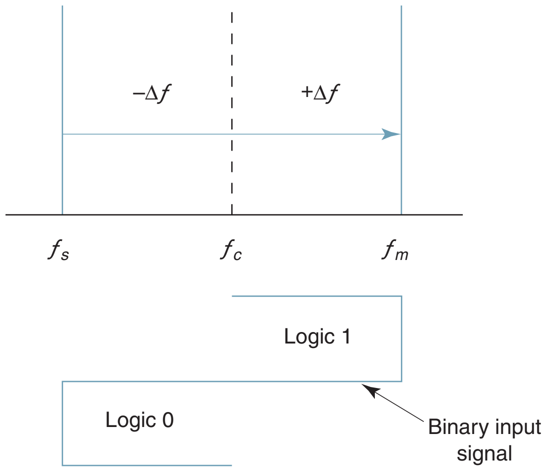


FIGURE 3 FSK in the frequency domain

is determined by the polarity. The modulating signal is a normalized binary waveform where a logic 1 = +1 V and a logic 0 = −1 V. Thus, for a logic 1 input, $v_m(t) = +1$, Equation 13 can be rewritten as

$$v_{fsk}(t) = V_c \cos[2\pi(f_c + \Delta f)t]$$

For a logic 0 input, $v_m(t) = -1$, Equation 13 becomes

$$v_{fsk}(t) = V_c \cos[2\pi(f_c - \Delta f)t]$$

With binary FSK, the carrier center frequency (f_c) is shifted (deviated) up and down in the frequency domain by the binary input signal as shown in Figure 3. As the binary input signal changes from a logic 0 to a logic 1 and vice versa, the output frequency shifts between two frequencies: a mark, or logic 1 frequency (f_m), and a space, or logic 0 frequency (f_s). The mark and space frequencies are separated from the carrier frequency by the peak frequency deviation (Δf) and from each other by 2 Δf.

With FSK, frequency deviation is defined as the difference between either the mark or space frequency and the center frequency, or half the difference between the mark and space frequencies. Frequency deviation is illustrated in Figure 3 and expressed mathematically as

$$\Delta f = \frac{|f_m - f_s|}{2} \tag{14}$$

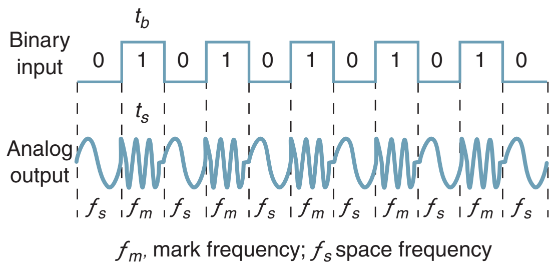
where Δf = frequency deviation (hertz)

$|f_m - f_s|$ = absolute difference between the mark and space frequencies (hertz)

Figure 4a shows in the time domain the binary input to an FSK modulator and the corresponding FSK output. As the figure shows, when the binary input (f_b) changes from a logic 1 to a logic 0 and vice versa, the FSK output frequency shifts from a mark (f_m) to a space (f_s) frequency and vice versa. In Figure 4a, the mark frequency is the higher frequency ($f_c + \Delta f$), and the space frequency is the lower frequency ($f_c - \Delta f$), although this relationship could be just the opposite. Figure 4b shows the truth table for a binary FSK modulator. The truth table shows the input and output possibilities for a given digital modulation scheme.

4-1 FSK Bit Rate, Baud, and Bandwidth

In Figure 4a, it can be seen that the time of one bit (t_b) is the same as the time the FSK output is a mark of space frequency (t_s). Thus, the bit time equals the time of an FSK signaling element, and the bit rate equals the baud.



(a)

binary input	frequency output
0	space (f_s)
1	mark (f_m)

(b)

FIGURE 4 FSK in the time domain: (a) waveform; (b) truth table

The baud for binary FSK can also be determined by substituting $N = 1$ in Equation 11:

$$\text{baud} = \frac{f_b}{1} = f_b$$

FSK is the exception to the rule for digital modulation, as the minimum bandwidth is not determined from Equation 10. The minimum bandwidth for FSK is given as

$$B = |(f_s - f_b) - (f_m - f_b)|$$
$$= |f_s - f_m| + 2f_b$$

and since $|f_s - f_m|$ equals $2\Delta f$, the minimum bandwidth can be approximated as

$$B = 2(\Delta f + f_b) \tag{15}$$

where B = minimum Nyquist bandwidth (hertz)
Δf = frequency deviation ($|f_m - f_s|$) (hertz)
f_b = input bit rate (bps)

Note how closely Equation 15 resembles Carson's rule for determining the approximate bandwidth for an FM wave. The only difference in the two equations is that, for FSK, the bit rate (f_b) is substituted for the modulating-signal frequency (f_m).

Example 2

Determine (a) the peak frequency deviation, (b) minimum bandwidth, and (c) baud for a binary FSK signal with a mark frequency of 49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps.

Solution **a.** The peak frequency deviation is determined from Equation 14:

$$\Delta f = \frac{|49\text{kHz} - 51\text{kHz}|}{2}$$
$$= 1 \text{ kHz}$$

b. The minimum bandwidth is determined from Equation 15:

$$B = 2(1000 + 2000)$$
$$= 6 \text{ kHz}$$

c. For FSK, $N = 1$, and the baud is determined from Equation 11 as

$$\text{baud} = \frac{2000}{1} = 2000$$

Bessel functions can also be used to determine the approximate bandwidth for an FSK wave. As shown in Figure 5, the fastest rate of change (highest fundamental frequency) in a nonreturn-to-zero (NRZ) binary signal occurs when alternating 1s and 0s are occurring (i.e., a square wave). Since it takes a high and a low to produce a cycle, the highest fundamental frequency present in a square wave equals the repetition rate of the square wave, which with a binary signal is equal to half the bit rate. Therefore,

$$f_a = \frac{f_b}{2} \tag{16}$$

where f_a = highest fundamental frequency of the binary input signal (hertz)
f_b = input bit rate (bps)

The formula used for modulation index in FM is also valid for FSK; thus,

$$h = \frac{\Delta f}{f_a} \qquad \text{(unitless)} \tag{17}$$

where h = FM modulation index called the h-factor in FSK
f_a = fundamental frequency of the binary modulating signal (hertz)
Δf = peak frequency deviation (hertz)

The worst-case modulation index (deviation ratio) is that which yields the widest bandwidth. The worst-case or widest bandwidth occurs when both the frequency deviation and the modulating-signal frequency are at their maximum values. As described earlier, the peak frequency deviation in FSK is constant and always at its maximum value, and the highest fundamental frequency is equal to half the incoming bit rate. Thus,

$$h = \frac{\dfrac{|f_m - f_s|}{2}}{\dfrac{f_b}{2}} \qquad \text{(unitless)}$$

or

$$h = \frac{|f_m - f_s|}{f_b} \tag{18}$$

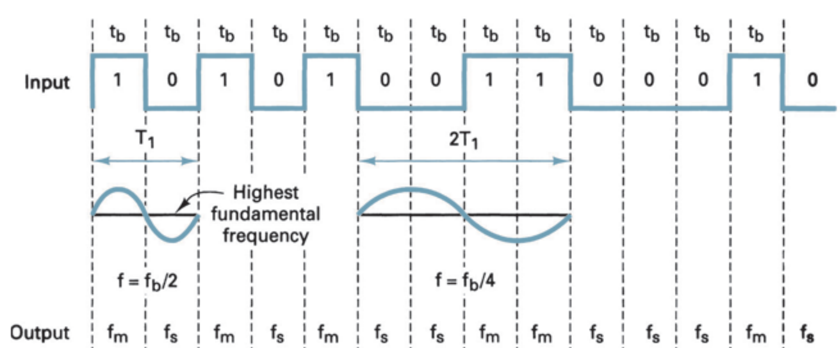


FIGURE 5 FSK modulator, t_b, time of one bit = $1/f_b$; f_m, mark frequency; f_s, space frequency; T_1, period of shortest cycle; $1/T_1$, fundamental frequency of binary square wave; f_b, input bit rate (bps)

where h = h-factor (unitless)
f_m = mark frequency (hertz)
f_s = space frequency (hertz)
f_b = bit rate (bits per second)

Example 3

Using a Bessel table, determine the minimum bandwidth for the same FSK signal described in Example 1 with a mark frequency of 49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps.

Solution The modulation index is found by substituting into Equation 17:

or
$$h = \frac{|49 \text{ kHz} - 51 \text{ kHz}|}{2 \text{ kbps}}$$
$$= \frac{2 \text{ kHz}}{2 \text{ kbps}}$$
$$= 1$$

From a Bessel table, three sets of significant sidebands are produced for a modulation index of one. Therefore, the bandwidth can be determined as follows:

$$B = 2(3 \times 1000)$$
$$= 6000 \text{ Hz}$$

The bandwidth determined in Example 3 using the Bessel table is identical to the bandwidth determined in Example 2.

4-2 FSK Transmitter

Figure 6 shows a simplified binary FSK modulator, which is very similar to a conventional FM modulator and is very often a voltage-controlled oscillator (VCO). The center frequency (f_c) is chosen such that it falls halfway between the mark and space frequencies. A logic 1 input shifts the VCO output to the mark frequency, and a logic 0 input shifts the VCO output to the space frequency. Consequently, as the binary input signal changes back and forth between logic 1 and logic 0 conditions, the VCO output shifts or deviates back and forth between the mark and space frequencies.

In a binary FSK modulator, Δf is the peak frequency deviation of the carrier and is equal to the difference between the carrier rest frequency and either the mark or the space frequency (or half the difference between the carrier rest frequency) and either the mark or the space frequency (or half the difference between the mark and space frequencies). A VCO-FSK modulator can be operated in the sweep mode where the peak frequency deviation is

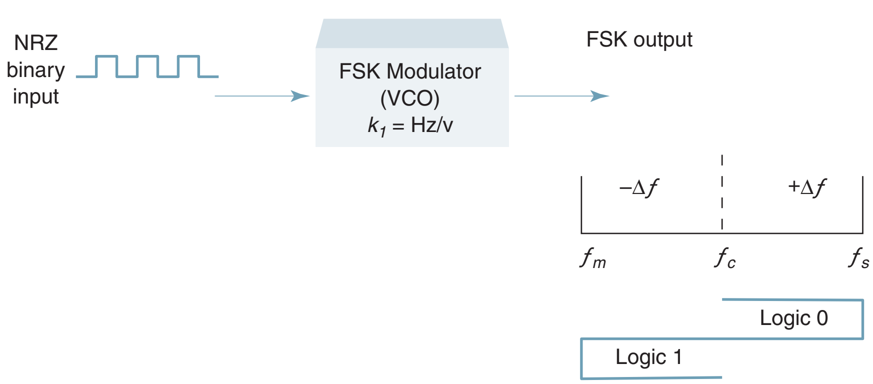


FIGURE 6 FSK modulator

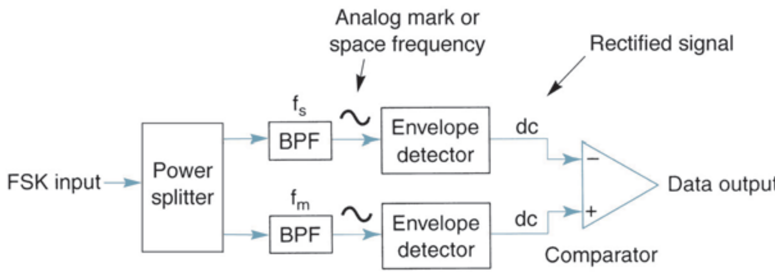


FIGURE 7 Noncoherent FSK demodulator

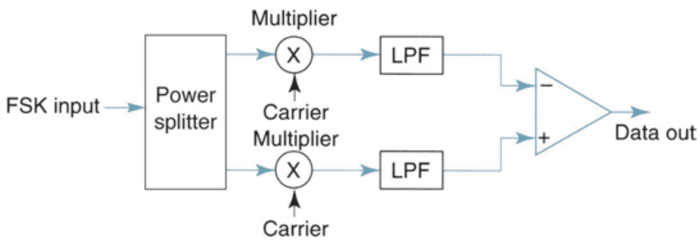


FIGURE 8 Coherent FSK demodulator

simply the product of the binary input voltage and the deviation sensitivity of the VCO. With the sweep mode of modulation, the frequency deviation is expressed mathematically as

$$\Delta f = v_m(t)k_l \tag{19}$$

where Δf = peak frequency deviation (hertz)
$v_m(t)$ = peak binary modulating-signal voltage (volts)
k_l = deviation sensitivity (hertz per volt).

With binary FSK, the amplitude of the input signal can only be one of two values, one for a logic 1 condition and one for a logic 0 condition. Therefore, the peak frequency deviation is constant and always at its maximum value. Frequency deviation is simply plus or minus the peak voltage of the binary signal times the deviation sensitivity of the VCO. Since the peak voltage is the same for a logic 1 as it is for a logic 0, the magnitude of the frequency deviation is also the same for a logic 1 as it is for a logic 0.

4-3 FSK Receiver

FSK demodulation is quite simple with a circuit such as the one shown in Figure 7. The FSK input signal is simultaneously applied to the inputs of both bandpass filters (BPFs) through a power splitter. The respective filter passes only the mark or only the space frequency on to its respective envelope detector. The envelope detectors, in turn, indicate the total power in each passband, and the comparator responds to the largest of the two powers. This type of FSK detection is referred to as noncoherent detection; there is no frequency involved in the demodulation process that is synchronized either in phase, frequency, or both with the incoming FSK signal.

Figure 8 shows the block diagram for a coherent FSK receiver. The incoming FSK signal is multiplied by a recovered carrier signal that has the exact same frequency and phase as the transmitter reference. However, the two transmitted frequencies (the mark and space frequencies) are not generally continuous; it is not practical to reproduce a local reference that is coherent with both of them. Consequently, coherent FSK detection is seldom used.

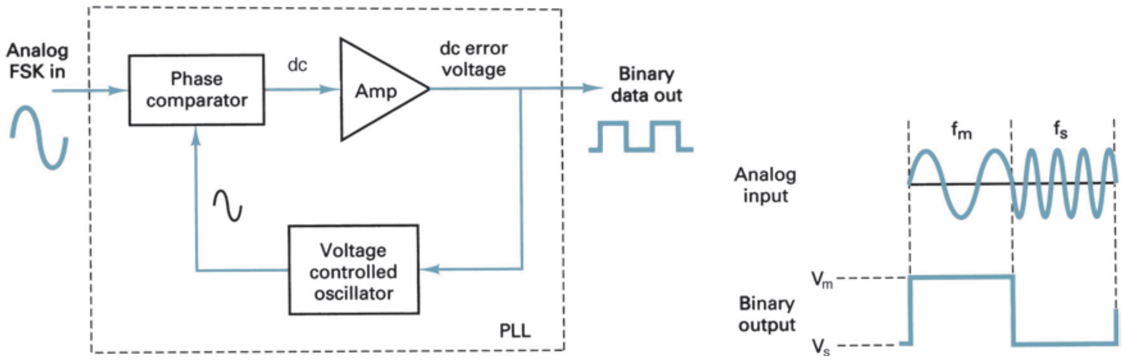


FIGURE 9 PLL-FSK demodulator

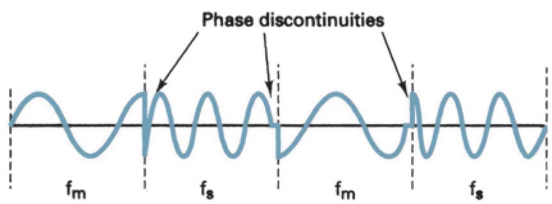


FIGURE 10 Noncontinuous FSK waveform

The most common circuit used for demodulating binary FSK signals is the *phase-locked loop* (PLL), which is shown in block diagram form in Figure 9. A PLL-FSK demodulator works similarly to a PLL-FM demodulator. As the input to the PLL shifts between the mark and space frequencies, the *dc error voltage* at the output of the phase comparator follows the frequency shift. Because there are only two input frequencies (mark and space), there are also only two output error voltages. One represents a logic 1 and the other a logic 0. Therefore, the output is a two-level (binary) representation of the FSK input. Generally, the natural frequency of the PLL is made equal to the center frequency of the FSK modulator. As a result, the changes in the dc error voltage follow the changes in the analog input frequency and are symmetrical around 0 V.

Binary FSK has a poorer error performance than PSK or QAM and, consequently, is seldom used for high-performance digital radio systems. Its use is restricted to low-performance, low-cost, asynchronous data modems that are used for data communications over analog, voice-band telephone lines.

4-4 Continuous-Phase Frequency-Shift Keying

Continuous-phase frequency-shift keying (CP-FSK) is binary FSK except the mark and space frequencies are synchronized with the input binary bit rate. Synchronous simply implies that there is a precise time relationship between the two; it does not mean they are equal. With CP-FSK, the mark and space frequencies are selected such that they are separated from the center frequency by an exact multiple of one-half the bit rate (f_m and $f_s = n[f_b/2]$), where n = any integer). This ensures a smooth phase transition in the analog output signal when it changes from a mark to a space frequency or vice versa. Figure 10 shows a noncontinuous FSK waveform. It can be seen that when the input changes from a logic 1 to a logic 0 and vice versa, there is an abrupt phase discontinuity in the analog signal. When this occurs, the demodulator has trouble following the frequency shift; consequently, an error may occur.

Figure 11 shows a continuous phase FSK waveform. Notice that when the output frequency changes, it is a smooth, continuous transition. Consequently, there are no phase discontinuities. CP-FSK has a better bit-error performance than conventional binary FSK for a given signal-to-noise ratio. The disadvantage of CP-FSK is that it requires synchronization circuits and is, therefore, more expensive to implement.