

Business Mathematics and Statistics

NCWEB Hansraj Centre

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Learning Goals:

In this class we will discuss:

- Concept of Time Series
- Components of Time Series
- Additive and Multiplicative Models
- Trend Analysis
- Finding trend by Moving Average Method

DEFINITION AND UTILITY OF TIME SERIES ANALYSIS

“When quantitative data are arranged in the order of their occurrence, the resulting statistical series is called a time series”.

The analysis of time series is of great utility not only to research workers but also to economists, businessmen and scientists etc., for the following reasons:

- 1) It helps in understanding past behavior of the variables under study.
- 2) It facilitates in forecasting the future behavior with the help of the changes that have taken place in the past.
- 3) It helps in planning future course of action.
- 4) It helps in knowing current accomplishment.
- 5) It is helpful to make comparisons between different time series and to decide significant conclusions.

COMPONENTS OF TIME SERIES

There are four components which form the basis for ‘Time Series Analysis’.

Long-term causes : Secular Trend or Trend (T)

Short-term causes :

Regular : Cyclical (C)

: Seasonal (S)

Irregular or Random : Erratic (I)

1) Secular Trend or Trend (T):

The causes which affect the variable gradually and permanently are termed as “Long-Term Causes”. The examples of such causes are: increase in the rate of capital formation, technological innovations, the introduction of automation, changes in productivity, improved marketing etc. The effect of long term causes is reflected in the tendency of a behavior, to move in an upward or downward direction, termed as ‘Trend’ or ‘Secular Trend’. It reveals as to how the time series has behaved over the period under study.

Short-term causes

The causes which affect the variables for the time being only are labelled as ‘Short-Term Causes’. The short term causes are further divided into two parts, they are ‘Regular’ and ‘Irregular’. Regular causes are further divided into two parts, namely ‘cyclical causes’ and ‘seasonal causes’.

2) Cyclical (C):

The cyclical variations are also termed as business cycle fluctuations, as they influence the variable. A business cycle is composed of prosperity, recession, depression and recovery. The periodic movements from prosperity to depression and back again to prosperity vary both in time and intensity.

3) Seasonal (S):

The seasonal causes, like weather conditions, business climate and even local customs and ceremonies together play an important role in giving rise to seasonal movements in almost all the business activities. For instance the yearly weather conditions directly affect agricultural production and marketing.

4) Irregular or Random:

Irregular causes are also termed as ‘Erratic’ or ‘Random’ causes. Random variations are caused by infrequent occurrences such as wars, strikes, earthquakes, floods etc. These reasons either go very deep downwards or very high upwards.

Mathematical Statement of the Composition of Time Series

A time series may not be affected by all type of variations. Some of these types of variations may affect a few time series, while the other series may be affected by all of them. Hence in analysing time series, these effects are isolated. In classical time series analysis it is assumed that any given observation is made up of trend, seasonal, cyclical and irregular movements and these four components have multiplicative relationship.

Multiplicative Model:

According to this model, a time series is the product of its four components. This is the most commonly used model in the decomposition of time series.

Symbolically: $O = T \times S \times C \times I$

where O refers to original data,

T refers to trend,

S refers to seasonal variations.

C refers to cyclical variation and

I refers to irregular variations.

Example: If in a multiplicative model $T=500$, $S=1.4$, $C=1.20$ and $I= 0.7$ then

$O=T \times S \times C \times I$, by substituting the values we get

$$O = 500 \times 1.4 \times 1.20 \times 0.7 = 608$$

Additive Model: There is another model called Additive model in which a particular observation in a time series is the sum of these four components.

$$O = T + S + C + I$$

Example: In additive model, $T=500$, $S=100$, $C= 25$, $I= -50$ then

$$O = 500 + 100 + 25 - 50 = 575$$

In business research, normally the multiplicative model is more suited and used more frequently for the purposes of analysis to time series. Because, the data related to business and economic time series is the result of interaction of a number of factors which individually cannot be held responsible for generating any specific type of variations.

Methods of Measuring Trend

Trend can be determined by: (i) moving averages method and (ii) least-squares method. They are explained below.

Moving Average Method

While considering matters such as trend of prices, sales, profits, etc., a particular type of average known as moving average is used. It is a measure of trend (long-term tendency of the data) in the time series data. Moving average is an arithmetic average of data arising over a period of time and is calculated **by replacing the first item in the average by the newly arising item.**

Odd series:

The process of successively averaging, say three years data and establishing each average as the moving average value of the central year in the group, should be carried throughout the entire series. For a five item, seven item or other moving averages, the same procedure is followed: the average obtained each time being considered as representative of the middle period of the group

Even Series:

If the moving average covers an even number of years, each average will still be representative of the midpoint of the period covered, but this mid-point will fall half way between the two middle years. In the case of a four year moving average, for instance each average represents a point half way between the second and third years. In such a case a second moving average may be used to 'recentre' the averages. If the first moving averages gives averages centering half-way between the years, a further two-point moving average will recentre the data exactly on the years.

Computation

In the computation of moving average, the period of moving average is a very important factor. For example, for yearly values A, B, C, D, E, and F, the three yearly moving averages can be computed as shown in Table 1.

Table 1: Computation of Moving Averages

Yearly Values	3 Yearly Moving Totals	3 Yearly Moving Averages
A
B	(A+B+C)	(A+B+C)/3
C	(B+C+D)	(B+C+D)/3
D	(C+D+E)	(C+D+E)/3
E	(D+E+F)	(D+E+F)/3
F

3 year Moving Averages:

Years	Sales (*000 tonnes)	3 Yearly Moving Totals
2001	15	--
2002	25	72
2003	32	81
2004	24	75
2005	19	60
2006	17	--

Please note that the moving average for the first three years (2001, 2002 and 2003) i.e., 72 is associated with the middle year 2002. Having dropped the first year, the moving average of the next three years i.e. 2002, 2003 and 2004 is placed against 2003; and so on. You must also note that moving average for the first year and the last year in the given data cannot be obtained.

4 Yearly Moving Average:

Illustration : Compute 4 yearly moving averages for the following data:

Years	:	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sales (Rs. In: '000)		75	60	54	69	86	65	63	80	90	72

Solution

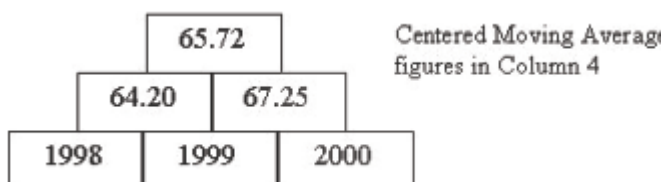
Year	Sales (Rs. In 000s)	4 Yearly Moving Total	4 Yearly Moving Average	4 Yearly Moving Average Centered
1997	75	--	--	--
1998	60	--	--	--
1999	54	258	64.20	--
2000	69	269	67.25	67.88
2001	86	274	68.50	69.62
2002	65	283	70.75	72.12
2003	63	294	73.50	73.50
2004	80	298	74.50	75.37
2005	90	305	76.25	--
2006	72	--	--	--

Explanation:

The total 258 of the first four figures (years 1997 to 2000) and their average 64.20 is written against the middle of this time period i.e., middle of the years 1998 and 1999. This middle time period is a specially designed year taking last six months from 1998 and the first six months from 1999. Similarly, the total 269 corresponding to year 1998 to 2001 and their average 67.25 is written against the specially designed year i.e., the mid-year of 1999 and 2000. This process continues till the last average 76.25 and the total 305 is noted against the mid-year of 2004 and 2005.

Centred moving average

To find out the first centred moving average 65.72 (i.e., a figure of moving average which will coincide with the year 1999), we have to find the mid-value 64.20 and 67.25, the first two figures in Column 4. This can be easily seen with the help of diagram given below:



The diagram shows that the figure which coincides with the year 1999 will come from half of 64.20 and half of 67.25, which means that it is the mean of the two moving averages. This mean value 65.72 is, therefore, called centered moving average and is entered in the last column. The various entered moving averages are, thus, calculated by taking successively mean of the two consecutive figures from Column 4.

5 Year Moving Average:

Illustration: Calculate 5-yearly moving average trend for the time series given below.

Year	: 1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Quantity	: 239	242	238	252	257	250	273	270	268	288	284
Year	: 1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	
Quantity	: 282	300	303	298	313	317	309	329	333	327	

Solution:

Year	Quantity	5-yearly moving total	5-yearly moving average
1980	239		
1981	242		
1982	238	1228	245.6
1983	252	1239	247.8
1984	257	1270	254.0
1985	250	1302	260.4
1986	273	1318	263.6
1987	270	1349	269.8
1988	268	1383	276.6
1989	288	1392	278.4
1990	284	1422	284.4
1991	282	1457	291.4
1992	300	1467	293.4
1993	303	1496	299.2
1994	298	1531	306.2
1995	313	1540	308.0
1996	317	1566	313.2
1997	309	1601	320.2
1998	329	1615	323.0
1999	333		
2000	327		

Explanation:

Obtain the total of first five years data. Find out the difference between the first and sixth term and add to the total to obtain the total of second to sixth term. In this way the difference between the term to be omitted and the term to be included is added to the preceding total in order to obtain the next successive total.

Merits

1. This is a very simple method.
2. The element of flexibility is always present in this method as all the calculations have not to be altered if same data is added. It only provides additional trend values.
3. If there is a coincidence of the period of moving averages and the period of cyclical fluctuations, the fluctuations automatically disappear.
4. The pattern of moving average is determined in the trend of data and remains unaffected by the choice of method to be employed.
5. It can be put to utmost use in case of series having strikingly irregular trend

Limitations

- I. It is not possible to have a trend value for each and every year. As the period of moving average increases, there is always an increase in the number of years for which trend values cannot be calculated and known. For example, in a five yearly moving average, trend value cannot be obtained for the first two years and last two years, in a seven yearly moving average for the first three years and last three years and so on. But usually values of the extreme years are of great interest.
2. There is no hard and fast rule for the selection of a period of moving average.
3. Forecasting is one of the leading objectives of trend analysis. But this objective remains unfulfilled because moving average is not represented by a mathematical function.
4. Theoretically it is claimed that cyclical fluctuations are ironed out if period of moving average coincide with period of cycle, but in practice cycles are not perfectly periodic.