Statistical mechanics

Lecture note: 1

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Topics covered in this Lecture:

1. Revision
2. Completely degenerate F-D system (T=0) :
   - find the expression for N
   - Fermi energy
   - Internal energy
   - Fermi temperature
   - Mean energy of fermions
   - Pressure of Fermions
3. Electron gas in metals
4. Tutorial sheet.

Revision:

- The occupation no. for the i[th] energy level is given as

$$n = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} + 1}$$

- For the case of energy levels which are closely spaced, Fermi distribution function is given as

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon - \varepsilon_f}{kT}} + 1}$$

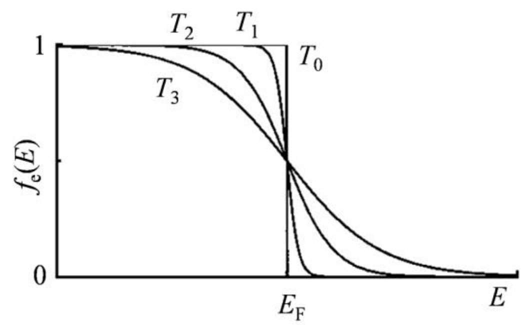

Figure 1: Representation of Fermi distribution function at various temperature.

- For $\varepsilon = \varepsilon_{f,}$ $f(\varepsilon) = 1/2$ at all temperature and at T = 0, all states above ε_f are completely empty and all state below ε_f are fully occupied. As the temperature increases the fermions occupied to higher energy states.

- No of state is given by g(p)dp

- Fugacity, A = exp(μ/kT)

- Total internal energy of a non-relativistic fermion system is greater than the classical energy.

- Fermi temperature $T_f = \varepsilon_f / k$

Completely degenerate F-D system (T=0)

Q. what do you mean by degenerate system?

Ans. This means how much the system deviates from the classical system.

From Fermi Dirac statistics, the occupation no. for the i[th] energy level is given as

$$n = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} + 1}$$

The above expression is when we consider that the energy levels are discrete. For macroscopic system, energy levels are discrete but spaced so closely, that we consider them as continuous distribution and for that case the above equation can be rewritten as

$$n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon - \mu}{kT}} + 1} \qquad (1)$$

No. of quantum states within energy range ε and ε+dε is given as:

$$g(\varepsilon)d\varepsilon = \frac{2\pi V (2m)^{\frac{3}{2}} \varepsilon^{1/2} d\varepsilon}{h^3} \qquad (2)$$

Since fermions cans be distinguishable on the basis on their intrinsic angular momentum, thus we have to consider spin degeneracy also and for that multiply equation 2 by g_s

$$g(\varepsilon)d\varepsilon = \frac{g_s 2\pi V (2m)^{\frac{3}{2}} \varepsilon^{1/2} d\varepsilon}{h^3} \qquad (3)$$

No. of particle within energy range ε and ε+dε are given as by multiplying eqn. 3 and eqn. 1

$$n(\varepsilon)d\varepsilon = \frac{g_s 2\pi V (2m)^{\frac{3}{2}} \varepsilon^{1/2} d\varepsilon}{h^3} \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} + 1} \qquad (4)$$

Total no. of particles are given as

$$N = \int_0^\infty \frac{g_s 2\pi V (2m)^{\frac{3}{2}} \varepsilon^{1/2}}{h^3} \frac{d\varepsilon}{e^{\frac{\varepsilon_i - \mu}{kT}} + 1} \qquad (5)$$

$$N = CV \int_0^\infty n(\varepsilon) \varepsilon^{1/2} d\varepsilon \qquad (6)$$

Where $C = \frac{g_s 2\pi (2m)^{\frac{3}{2}}}{h^3}$. Eqn. 6 gives no. of particles and since FD stat. follow pauli exclusion principle and total no of particles are N, so we can say that the lowest energy states upto N are completely occupied at T=0 K. Eqn. 6 can be further solved by considering the behavior of n(ε) at T = 0 K.

n(ε) = 0 if ε > μ and

n(ε) = 1 if ε < μ.

This means that at T = 0 K up to the energy ε = μ are occupied and the rest are completely empty. So the highest filled energy is called Fermi energy and ε = μ = ε_f. so up to energy level 0 to ε_f, n(ε) is equal to one and thus limits of integration in equation 6 are changed to 0 to ε_f.

$$N = CV \int_0^{\varepsilon_f} \varepsilon^{1/2} d\varepsilon \qquad (7)$$

$$N = \frac{2}{3} CV \varepsilon_f^{\frac{3}{2}} \qquad (8)$$

From eqn. 8 and C = $\frac{g_s 2\pi (2m)^{\frac{3}{2}}}{h^3}$, we will get the expression of **Fermi energy** as

$$\varepsilon_f = \frac{h^2}{2m} \left(\frac{3N}{4\pi g_s V}\right)^{\frac{2}{3}} \qquad (9)$$

Fermi Temperature is given as

$$T_f = \frac{\varepsilon_f}{k} = \frac{h^2}{2mk} \left(\frac{3N}{4\pi g_s V}\right)^{\frac{2}{3}} \qquad (10)$$

Total internal energy can be calculated as

$$E_O = CV \int_0^{\varepsilon_f} \varepsilon^{3/2} d\varepsilon \qquad (11)$$

$$E_O = \frac{2}{5} CV \varepsilon_f^{\frac{5}{2}} \qquad (12)$$

From eqn. 8 and 12.

$$E_O = \frac{3}{5} N \varepsilon_f^{\frac{5}{2}} \qquad (13)$$

Mean energy per fermion can be written as

$$\bar{\varepsilon} = \frac{E_O}{N} = \frac{3}{5} \varepsilon_f^{\frac{5}{2}} \qquad (14)$$

Heat capacity of fermions are given as C_V = (dE/dT) and since total internal energy is independent of temperature, thus specific heat capacity of Fermions is zero at T = 0.

Pressure of fermions at T = 0 K can be evaluated by using the expression P = 2E/3V. This gives pressure exerted by fermions at absolute zero

$$P_f = \frac{4C}{15} \varepsilon_f^{\frac{5}{2}} \qquad (15)$$

Although it is surprising to note that pressure is note equal to zero even at T = 0 K.

Electron gas in Metals:

The electrons in metals are assumed to move freely just like particles of gas. Thus electrons in metals can be assumed as electron gas. However, as per Sommerfled, electrons in metal are not free but they are bound and move under some uniform potential. Since electrons are assumed to obey Pauli Exclusion Principle, they should obey Fermi Dirac Statistics.

For electrons s =1/2 and thus g_s = 2S+1 = 2

Following eqn. 9, the Fermi energy for electrons can be calculated as

$$\varepsilon_f = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}} \qquad (16)$$

$$\varepsilon_f = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{\frac{2}{3}}$$

Fermi temperature can be written as:

$$T_f = \frac{\varepsilon_f}{k} = \frac{h^2}{8mk}\left(\frac{3N}{\pi V}\right)^{\frac{2}{3}} \qquad (17)$$

References:

1. *Thermal Physics by S.C. Garg, R.M. Bansal and C. K. Ghosh.*
2. *Statistical Physics, Berkley series volume 5.*
3. *Fundaments of Statitical Mechanics and Thermal Physics, F. Rief.*

Tutorial questions:

Q1. Calculate the Fermi energy and pressure at absolute zero in aluminum. The density of Al is $2.7*10^3$ kg m^{-3} ,atomic weight of Al is 26.98 kg $(kmol)^{-1}$. Take the standard values of h,N_a and $m_{e.}$

Q2. Calculate the Fermi energy and the electronic contribution to constant heat capacity of copper at 300 K. Take N/V = $8.5*10^{28}$ m^{-3} . Take the standard values of h,$N_{a,}$k, R and $m_{e.}$

Q3. Six fermions are attached in two compartments. The first compartments have 7 cells and second compartment has 8 cells of equal size. What is the total no. of microstate for the macrostate of (2,4).

Q4. Calculate the Fermi energy of silver, given that its density is 10.5g/cc. The atomic mass of silver is 108 g. Assume there is one free electron per atom.